CHAPTER 8

Quadratic Functions and Equations

Section 8A

Quadratic Functions

- 8-1 Identifying Quadratic Functions
- Algebra Lab Explore the Axis of Symmetry
- 8-2 Characteristics of Quadratic Functions
- 8-3 Graphing Quadratic Functions
- Technology Lab The Family of Quadratic Functions
- 8-4 Transforming Quadratic Functions

Section 8B

Solving Quadratic Equations

- 8-5 Solving Quadratic Equations by Graphing
- Technology Lab Explore Roots, Zeros, and *x*-Intercepts
- 8-6 Solving Quadratic Equations by Factoring
- 8-7 Solving Quadratic Equations by Using Square Roots
- Algebra Lab Model Completing the Square
- 8-8 Completing the Square
- 8-9 The Quadratic Formula and the Discriminant
- 8-10 Nonlinear Systems
- Extension Cubic Functions and Equations

Pacing Guide for 45-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 8

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
8-1 Lesson	Algebra Lab 8-2 Lesson	8-2 Lesson	8-3 Lesson	Technology Lab 8-4 Lesson
DAY 6	**DAY 7**	**DAY 8**	**DAY 9**	**DAY 10**
8-4 Lesson Ready to Go On?	Focus on Problem Solving 8-5 Lesson	8-5 Lesson Technology Lab	8-6 Lesson	8-6 Lesson 8-7 Lesson
DAY 11	**DAY 12**	**DAY 13**	**DAY 14**	**DAY 15**
8-7 Lesson Algebra Lab	8-8 Lesson	8-9 Lesson	8-10 Lesson	8-10 Lesson
DAY 16	**DAY 17**	**DAY 18**		
Multi-Step Test Prep Ready to Go On?	Extension	Chapter Review	Chapter Test	

Pacing Guide for 90-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 8

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
8-1 Lesson Algebra Lab 8-2 Lesson	8-2 Lesson 8-3 Lesson	Technology Lab 8-4 Lesson Multi-Step Test Prep	Ready to Go On? 8-5 Lesson Technology Lab	8-6 Lesson 8-7 Lesson
DAY 6	**DAY 7**	**DAY 8**	**DAY 9**	
8-7 Lesson Algebra Lab 8-8 Lesson	8-9 Lesson 8-10 Lesson	Multi-Step Test Prep Ready to Go On? Extension	Chapter Review Chapter Test	

ONGOING ASSESSMENT and INTERVENTION

	DIAGNOSE	PRESCRIBE
Assess Prior Knowledge	**Before the Chapter**	
	Diagnose readiness for the chapter.	Prescribe intervention.
	Are You Ready? SE	***Are You Ready? Intervention***
Formative Assessment	**Before Every Lesson**	
	Diagnose readiness for the lesson.	Prescribe intervention.
	Warm Up TE	**Reteach** CRB
	During Every Lesson	
	Diagnose understanding of lesson concepts.	Prescribe intervention.
	Check It Out! SE	**Reading Strategies** CRB
	Questioning Strategies TE	***Success for Every Learner***
	Think and Discuss SE	***Lesson Tutorial Videos***
	Write About It SE	
	Journal TE	
	After Every Lesson	
	Diagnose mastery of lesson concepts.	Prescribe intervention.
	Lesson Quiz TE	**Reteach** CRB
	Test Prep SE	**Test Prep Doctor** TE
	ExamView™ Assessment Suite	***Homework Help* Online**
	Before Chapter Testing	
	Diagnose mastery of concepts in chapter.	Prescribe intervention.
	Ready to Go On? SE	***Ready to Go On? Intervention***
	Multi-Step Test Prep SE	**Scaffolding Questions** TE
	Section Quizzes AR	**Reteach** CRB
	ExamView™ Assessment Suite	***Lesson Tutorial Videos***
	Before High Stakes Testing	
	Diagnose mastery of benchmark concepts.	Prescribe intervention.
	College Entrance Exam Practice SE	***College Entrance Exam Practice***
	Standardized Test Prep SE	
Summative Assessment	**After the Chapter**	
	Check mastery of chapter concepts.	Prescribe intervention.
	Multiple-Choice Tests (Forms A, B, C)	**Reteach** CRB
	Free-Response Tests (Forms A, B, C)	***Lesson Tutorial Videos***
	Performance Assessment AR	
	Cumulative Test AR	
	ExamView™ Assessment Suite	

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* **CRB** = *Chapter Resource Book* **AR** = *Assessment Resources* Available online

CHAPTER 8

Lesson Resources

Before the Lesson

Prepare

Teacher One Stop
- Editable lesson plans
- Calendar Planner
- Easy access to all chapter resources

Lesson Transparencies
- Teacher Tools

Teach the Lesson

Introduce

Alternate Openers: Explorations

Lesson Transparencies
- Warm Up
- Problem of the Day

Teach

Lesson Transparencies
- Teaching Transparencies

Know-It Notebook™
- Vocabulary
- Key Concepts

Power Presentations

Lesson Tutorial Videos SPANISH

Interactive Online Edition
- Lesson Activities
- Lesson Tutorial Videos

Lab Activities

Lab Resources Online

Online Interactivities SPANISH

TechKeys

Practice the Lesson

Practice

Chapter Resources
- Practice A, B, C

Practice and Problem Solving Workbook

IDEA Works!® Modified Worksheets and Tests

ExamView™ Assessment Suite

Homework Help Online

Online Interactivities SPANISH

Interactive Online Edition
- Homework Help

Apply

Chapter Resources
- Problem Solving

Practice and Problem Solving Workbook

Interactive Online Edition
- Chapter Project

Project Teacher Support

After the Lesson

Reteach

Chapter Resources
- Reteach
- Reading Strategies ELL

Success for Every Learner

Review

Interactive Answers and Solutions

Solutions Key

Know-It Notebook™
- Big Ideas
- Chapter Review

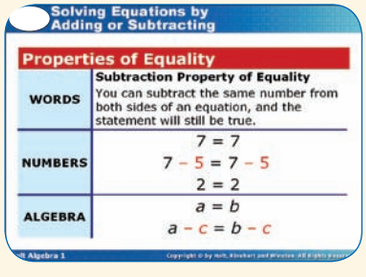
Extend

Chapter Resources
- Challenge

Technology Highlights for the Teacher

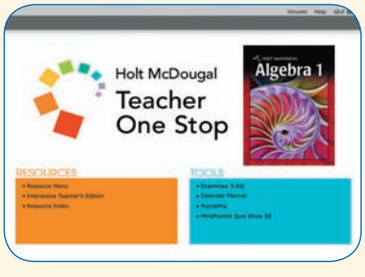
Power Presentations

Dynamic presentations to engage students. Complete PowerPoint® presentations for every lesson in this chapter.



Teacher One Stop SPANISH

Easy access to this chapter resources and assessments. Includes lesson planning software.



Interactive Online Edition

This chapter includes Tutorial Videos, Lesson Activities, Lesson Quizzes, Homework Help, and Chapter Project.

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* ELL English Language Learners SPANISH Spanish version available Available online

CHAPTER 8

Reaching All Learners

Teaching tips to help all learners appear throughout the chapter. A few that target specific students are included in the lists below.

All Learners

Lab Activities
Practice and Problem Solving Workbook
Know-It Notebook

Special Needs Students

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Inclusion ..... TE
IDEA Works!® Modified Worksheets and Tests
Ready to Go On? Intervention
Know-It Notebook
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

Developing Learners

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

On-Level Learners

Practice B ..... CRB
Problem Solving ..... CRB
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH

Advanced Learners

Practice C ..... CRB
Challenge ..... CRB
Challenge Exercises ..... SE
Reading and Writing Math *Extend* ..... TE
Are You Ready? Enrichment
Ready To Go On? Enrichment

English Language Learners

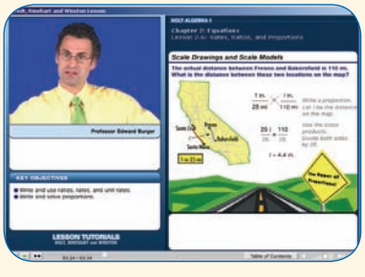
ENGLISH LANGUAGE LEARNERS

Reading Strategies ..... CRB
Are You Ready? Vocabulary ..... SE
Vocabulary Connections ..... SE
Vocabulary Review ..... SE
English Language Learners ..... TE
Success for Every Learner
Know-It Notebook
Multilingual Glossary
Lesson Tutorial Videos SPANISH

Technology Highlights for Reaching All Learners

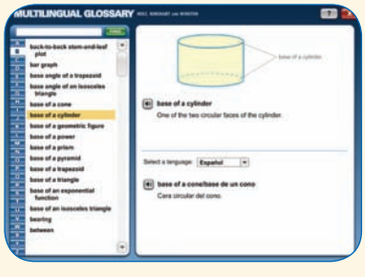
Lesson Tutorial Videos SPANISH

Starring Holt authors Ed Burger and Freddie Renfro! Live tutorials to support every lesson in this chapter.



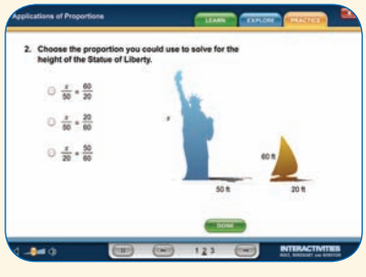
Multilingual Glossary

Searchable glossary includes definitions in English, Spanish, Vietnamese, Chinese, Hmong, Korean, and other languages.



Online Interactivities

Interactive tutorials provide visually engaging alternative opportunities to learn concepts and master skills.



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CHAPTER 8

Ongoing Assessment

Assessing Prior Knowledge

Determine whether students have the prerequisite concepts and skills for success in the chapter.

Are You Ready? ..... SE
Warm Up ..... TE

Test Preparation

Provide review and practice for the chapter and standardized tests.

Multi-Step Test Prep ..... SE
Study Guide: Review ..... SE
Test Tackler ..... SE
Standardized Test Prep ..... SE
College Entrance Exam Practice ..... SE
Countdown to Mastery ..... SE
***IDEA Works!*® Modified Worksheets and Tests**

Alternative Assessment

Assess students' understanding of the chapter concepts and combined problem-solving skills.

Chapter Project ..... SE
Alternative Assessment ..... TE
Performance Assessment ..... AR
Portfolio Assessment ..... AR

Lesson Assessment

Provide formative assessment for each lesson of the chapter.

Questioning Strategies ..... TE
Think and Discuss ..... SE
Check It Out! Exercises ..... SE
Write About It ..... SE
Journal ..... TE
Lesson Quiz ..... TE
Alternative Assessment ..... TE
***IDEA Works!*® Modified Worksheets and Tests**

Weekly Assessment

Provide formative assessment for each section of the chapter.

Multi-Step Test Prep ..... SE
Ready to Go On? ..... SE
Section Quizzes ..... AR
***ExamView*™ Assessment Suite**

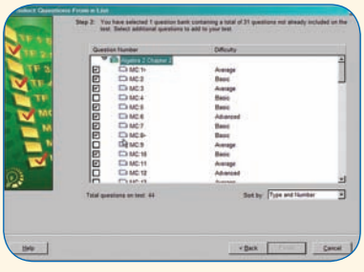
Chapter Assessment

Provide summative assessment of the chapter. mastery.

Chapter Test ..... SE
Chapter Test (Levels A, B, C) ..... AR
• Multiple Choice • Free Response
Cumulative Test ..... AR
***ExamView*™ Assessment Suite**
***IDEA Works!*® Modified Worksheets and Tests**

Technology Highlights for Assessment

ExamView™ Assessment Suite
Use the chapter problem banks to create assessments and worksheets to print out or deliver online. Includes dynamic problems.



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CHAPTER 8

Formal Assessment

Three levels (A, B, C) of multiple-choice and free-response chapter tests, along with a performance assessment, are available in the *Assessment Resources.*

A Chapter Test

C Chapter Test

MULTIPLE CHOICE

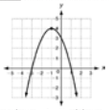
B Chapter Test

Select the best answer.

1. Which function is quadratic?

A	x	−4	−1	2	5
	y	18	3	6	27
B	x	−3	−2	−1	0
	y	−1	1	3	5
C	x	−2	0	2	4
	y	−13	−5	3	59
D	x	1	5	9	13
	y	2	−2	0	4

2. The vertex of this parabola shows that the _____ value of the function is _____.

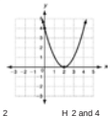


F maximum, −1 H minimum, −1
G maximum, 4 J minimum, 4

3. Which table of values would you use to graph $y = x^3 + 3$?

A	x	−2	−1	0	1	2
	y	−32	−1	0	1	32
B	x	−2	−1	0	1	2
	y	1	4	9	16	25
C	x	−2	−1	0	1	2
	y	7	4	3	4	7
D	x	−2	−1	0	1	2
	y	12	3	0	3	12

4. Find the zero(s) of $y = -x^2 - 4x + 4$ from its graph below.



F 2 H 2 and 4
G 4 J no zeros

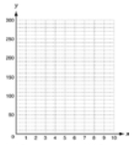
5. Find the axis of symmetry of the graph of $y = x^2 + 10x + 16$.

A $x = -10$ C $x = -5$
B $x = -8$ D $x = 13$

6. If you graph $y = 2x^2 - 8x - 10$, the y-intercept would be _____.

F −18 H 2
G −10 J 5

7. The height of a ball in feet is modeled by $f(x) = -16x^2 + 128x$, where x is the time in seconds after it is hit. How long is the ball in the air?



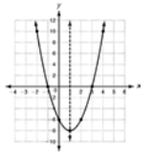
A 4 s C 128 s
B 8 s D 256 s

B Chapter Test *(continued)*

8. Compare the graph of $g(x) = 6x^2$ with the graph of $f(x) = x^2$.

F $g(x)$ is narrower.
G $g(x)$ is translated up.
H $g(x)$ is translated down.
J $g(x)$ is wider.

9. Use this graph of the quadratic function $y = 2x^2 - 4x - 6$ to solve the equation $2x^2 - 4x - 6 = 0$.



A $x = -2$ or $x = 4$ C $x = 0$ or $x = 2$
B $x = -1$ or $x = 3$ D $x = 1$

10. Solve $x^2 - 12 = -4x$ by factoring.

F $x = -2$ or $x = 6$ H $x = 2$ or $x = -6$
G $x = -3$ or $x = 4$ J $x = 3$ or $x = -4$

11. Solve the system by elimination.

$$\begin{cases} y = x^2 + 3x + 3 \\ x - y = -11 \end{cases}$$

A (−2, 1), (4, 31) C (−4, 7), (−2, 9)
B (−4, 7) (2, 13) D (2, 13), (4, 15)

12. Solve $9x^2 - 4 = 0$ using square roots.

F $x = \pm\frac{9}{4}$ H $x = \pm\frac{4}{9}$
G $x = \pm\frac{2}{3}$ J $x = \pm\frac{3}{2}$

13. Which number completes the square to form a perfect square trinomial?

$$x^2 + 7x + \square$$

A $\frac{7}{2}$ C $\frac{49}{2}$
B $\frac{49}{4}$ D 49

14. Solve $x^2 + 8x + 18 = 0$ by completing the square.

F $x = -2$ or $x = -6$
G $x = -4 + \sqrt{2}$ or $x = -4 - \sqrt{2}$
H $x = -4 + \sqrt{34}$ or $x = -4 - \sqrt{34}$
J no real solutions

15. Solve $3x^2 = 5x + 8$ using the Quadratic Formula.

A $x = -3$ or $x = 8$
B $x = -1$ or $x = -\frac{8}{3}$
C $x = \frac{5 \pm \sqrt{71}}{6}$
D no real solutions

16. Find the number of real solutions of the equation $x^2 + 10x + 35 = 0$ using the discriminant.

F 0 H 2
G 1 J 3

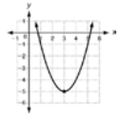
FREE RESPONSE

B Chapter Test

1. Tell whether this function is quadratic. Explain.

x	−2	0	2	4	
y	5	−1	−3	−1	

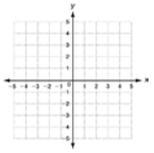
2. Identify the vertex of this parabola. Then give the minimum or maximum value of the function.



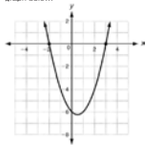
vertex: _____

3. Use a table of values to graph $y = x^2 - 4$.

x	−2	−1	0	1	2
y					



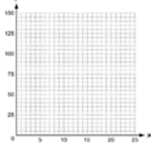
4. Find the zeros of $y = -x^2 - x - 6$ from its graph below.



5. Find the axis of symmetry of the graph of $y = x^2 - 12x + 8$.

6. If you graph $y = 5x^2 + 7x - 15$, what would be the y-intercept?

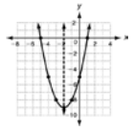
7. The height of a ball in meters is modeled by $f(x) = -5x^2 + 50x$, where x is the time in seconds after it is hit. How long is the ball in the air?



B Chapter Test *(continued)*

8. Compare the graph of $g(x) = \frac{1}{3}x^2$ with the graph of $f(x) = x^2$.

9. Use this graph of the quadratic function $y = x^2 + 4x - 5$ to solve the equation $x^2 + 4x - 5 = 0$.



10. Solve $x^2 + 18 = 11x$ by factoring.

11. Solve the system by elimination.

$$\begin{cases} y = x^2 + 2 \\ 4x - y = 2 \end{cases}$$

12. Solve $16x^2 - 1 = 0$ using square roots.

13. Complete the square to form a perfect square trinomial.

$$x^2 + 5x + ____$$

14. Solve $x^2 - 6x - 13 = 0$ by completing the square.

15. Solve $2x^2 + 7x = -8$ using the Quadratic Formula.

16. Find the number of real solutions of the equation $x^2 - 18x + 81 = 0$ using the discriminant.

PERFORMANCE ASSESSMENT

Chapter Test

Bunny Cage

Purpose

This performance task assesses the student's ability to graph a quadratic function, identify characteristics of its graph, solve quadratic equations and nonlinear systems of equations.

Time

40–50 minutes

Grouping

Groups of four

Preparation Hints

Review the characteristics of quadratic functions—zeros, axis of symmetry, and vertex—and how to use them to graph. Review these methods of solving quadratic equations—graphing, factoring, and using the quadratic formula. Review these methods of solving nonlinear systems of equations—substitution, elimination, and graphing.

Overview

Students first graph a quadratic function that represents the area of a rectangle given a fixed perimeter. When group members compare their results using different perimeters, they should see that a square always maximizes the area. Students then use their graphs and/or quadratic equations to find the dimensions of rectangles that give a desired area.

Introduce the Task

Emphasize that for questions 1–8, each member should be working with a single yet different measurement. If you prefer to assign measurements, 20 and 24 ft are less challenging, while 40 and 50 ft are more challenging.

Performance Indicators

_____ Defines the area of a rectangle as a quadratic function of its width.
_____ Graphs the quadratic function and identifies key characteristics.
_____ Interprets the vertex as the width that maximizes the area.
_____ Solves quadratic equations (graphically or algebraically) to find the dimensions necessary to create a given area.

Scoring Rubric

Level 4: Student solves individual questions correctly (1–8) and has thorough solutions for the group questions (9–10).
Level 3: Student solves most individual questions correctly and has minimal solutions for the group questions.
Level 2: Student solves a few individual questions and has minimal solutions for the group questions.
Level 1: Student has no solutions to any of the questions.

Chapter Test *(continued)*

Bunny Cage

First, each member of your group should pick one of these measurements and work individually to solve questions 1–8:

20 ft 24 ft 40 ft 50 ft

1. Complete the next sentence using the measurement you chose.
Erica has _____ of chain-link to make a rectangular cage for her pet rabbit. She will use all of the chain-link, so the perimeter of the cage will be the same as the entire length.
2. Let w represent the width of the cage in feet. Use w to write an expression for the length of the cage. (*Hint:* Use the perimeter formula, $P = 2l + 2w$, and solve for length.)
3. Use your answer to question 2 to write a quadratic function for the cage's area, A, as a function of width, w.
4. Graph your quadratic function on the grid at right.
5. Find the zeros of your function. What do they represent?
6. What is the axis of symmetry? How is it related to the amount of chain-link?
7. Find the vertex. What does it represent?
8. Assume Erica's goal is to maximize the area for her rabbit. What width and length should she use?

Share your results and work together as a group for questions 9–10.

9. Regardless of the amount of chain-link you work with, when you maximize the area of a rectangle, what shape do you get?
10. Assume Erica wants the area to be exactly 36 ft^2. For each amount of chain-link, find the dimensions of the rectangle that she should use. Support each answer with work or an explanation.
a. 20 feet
b. 24 feet
c. 40 feet
d. 50 feet

CHAPTER 8

MATHEMATICAL PRACTICES **Reason abstractly and quantitatively.**

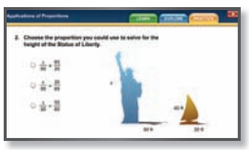
SECTION 8A
Quadratic Functions

MULTI-STEP TEST PREP Students write, solve, and graph equations to model real-world rocket-flight situations.

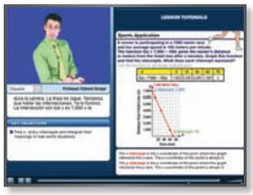
SECTION 8B
Solving Quadratic Equations

MULTI-STEP TEST PREP Students write, solve, and graph equations to model a real-world situation involving the motion of a golf ball.

Interactivities Online ▶



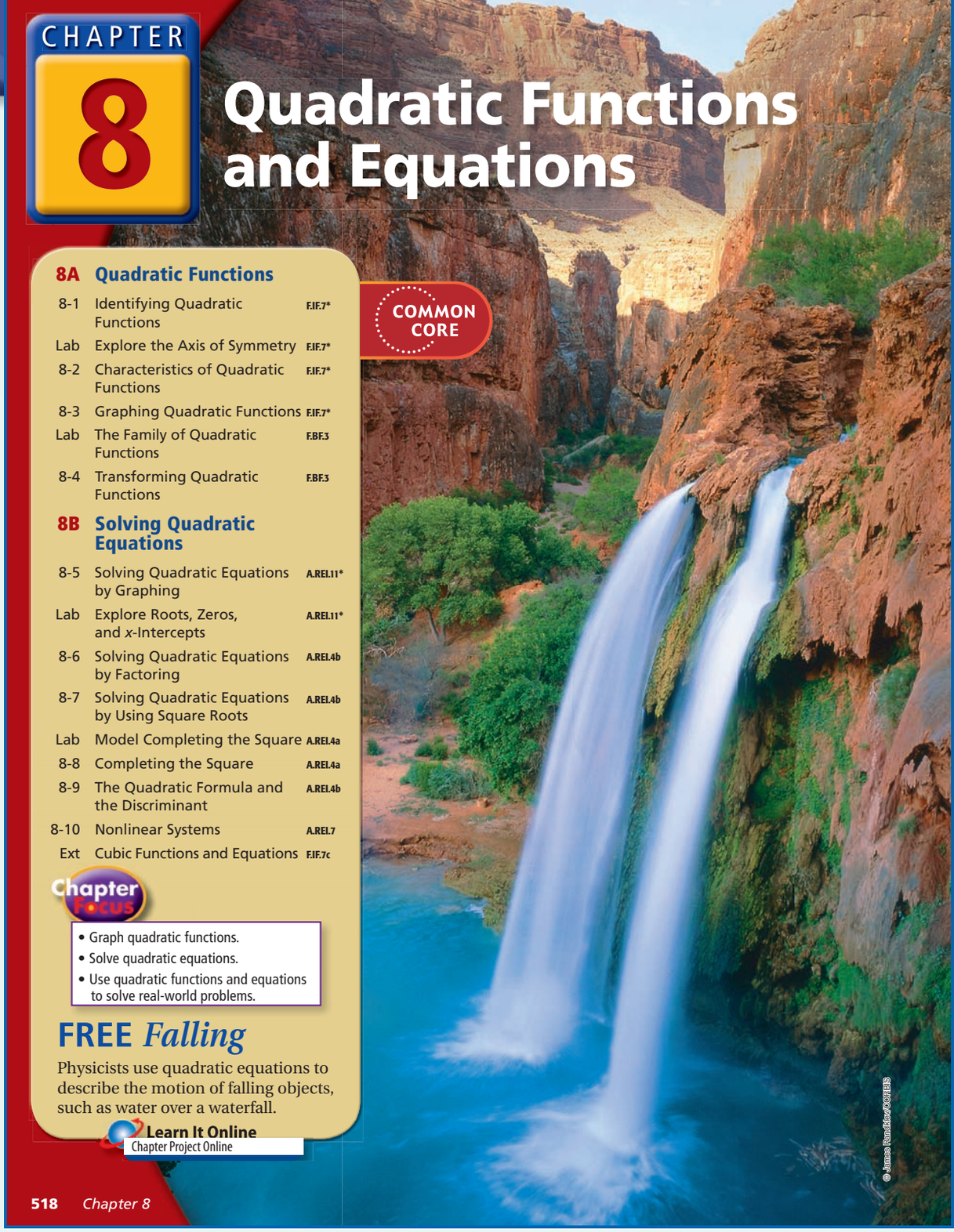
Video Lesson Tutorials Online



Lesson Tutorial Videos are available for EVERY example.

CHAPTER 8

Quadratic Functions and Equations



COMMON CORE

8A Quadratic Functions

8-1	Identifying Quadratic Functions	F.IF.7*
Lab	Explore the Axis of Symmetry	F.IF.7*
8-2	Characteristics of Quadratic Functions	F.IF.7*
8-3	Graphing Quadratic Functions	F.IF.7*
Lab	The Family of Quadratic Functions	F.BF.3
8-4	Transforming Quadratic Functions	F.BF.3

8B Solving Quadratic Equations

8-5	Solving Quadratic Equations by Graphing	A.REI.11*
Lab	Explore Roots, Zeros, and *x*-Intercepts	A.REI.11*
8-6	Solving Quadratic Equations by Factoring	A.REI.4b
8-7	Solving Quadratic Equations by Using Square Roots	A.REI.4b
Lab	Model Completing the Square	A.REI.4a
8-8	Completing the Square	A.REI.4a
8-9	The Quadratic Formula and the Discriminant	A.REI.4b
8-10	Nonlinear Systems	A.REI.7
Ext	Cubic Functions and Equations	F.IF.7c

Chapter Focus

- Graph quadratic functions.
- Solve quadratic equations.
- Use quadratic functions and equations to solve real-world problems.

FREE *Falling*

Physicists use quadratic equations to describe the motion of falling objects, such as water over a waterfall.

Learn It Online
Chapter Project Online

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Chapter Project

A project for this chapter is available online.

Project Resources

All project resources for teachers and students are provided online.

Learn It Online
Chapter Project Online

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. factoring **A**
2. quadratic **D**
3. trinomial **C**
4. x-intercept **B**

A. the process of writing a number or an algebraic expression as a product

B. the x-coordinate(s) of the point(s) where a graph intersects the x-axis

C. a polynomial with three terms

D. a polynomial with degree 2

E. the first number of an ordered pair of numbers that describes the location of a point on the coordinate plane

Graph Functions

Graph each function for the given domain.

5. $y = -2x + 8$; D: $\{-4, -2, 0, 2, 4\}$
6. $y = (x + 1)^2$; D: $\{-3, -2, -1, 0, 1\}$
7. $y = x^2 + 3$; D: $\{-2, -1, 0, 1, 2\}$
8. $y = 2x^2$; D: all real numbers

Multiply Binomials

Find each product.

9. $(m + 2)(m + 5)$ **$m^2 + 7m + 10$**
10. $(y - 7)(y + 2)$ **$y^2 - 5y - 14$**
11. $(2a + 4)(5a + 6)$ **$10a^2 + 32a + 24$**
12. $(x + 1)(x + 1)$ **$x^2 + 2x + 1$**
13. $(t + 5)(t + 5)$ **$t^2 + 10t + 25$**
14. $(3n - 8)(3n - 8)$ **$9n^2 - 48n + 64$**

Factor Trinomials

Factor each polynomial completely.

15. $x^2 - 2x + 1$ **$(x - 1)^2$**
16. $x^2 - x - 2$ **$(x - 2)(x + 1)$**
17. $x^2 - 6x + 5$ **$(x - 5)(x - 1)$**
18. $x^2 - x - 12$ **$(x - 4)(x + 3)$**
19. $x^2 - 9x + 18$ **$(x - 6)(x - 3)$**
20. $x^2 - 7x - 18$ **$(x - 9)(x + 2)$**

Squares and Square Roots

Find each square root.

21. $\sqrt{36}$ **6**
22. $\sqrt{121}$ **11**
23. $-\sqrt{64}$ **-8**
24. $\sqrt{16}\sqrt{81}$ **36**
25. $\sqrt{\frac{9}{25}}$ **$\frac{3}{5}$**
26. $-\sqrt{6(24)}$ **-12**

Solve Multi-Step Equations

Solve each equation.

27. $3m + 5 = 11$ **$m = 2$**
28. $3t + 4 = 10$ **$t = 2$**
29. $5n + 13 = 28$ **$n = 3$**
30. $2(k - 4) + k = 7$ **$k = 5$**
31. $10 = \frac{r}{3} + 8$ **$r = 6$**
32. $2(y - 6) = 8.6$ **$y = 10.3$**

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CHAPTER 8

ARE YOU READY?

Organizer

Objective: Assess students' understanding of prerequisite skills.

Assessing Prior Knowledge

INTERVENTION

Diagnose and Prescribe

Use this page to determine whether intervention is necessary or whether enrichment is appropriate.

Resources

- ***Are You Ready? Intervention and Enrichment* Worksheets**
- ***Are You Ready?* Online**

Answers

5–8. See Additional Answers.

ARE YOU READY?

Diagnose and Prescribe

NO INTERVENE

YES ENRICH

Prerequisite Skill	*Are You Ready? Intervention,* Chapter 8		
	Worksheets	Online	
Graph Functions	Skill 80	Activity 80	Diagnose and Prescribe Online
Multiply Binomials	Skill 64	Activity 64	
Factor Trinomials	Skill 67	Activity 67	
Squares and Square Roots	Skill 6	Activity 6	
Solve Multi-Step Equations	Skill 69	Activity 69	

***Are You Ready? Enrichment,* Chapter 8**
Worksheets
Online

Organizer

Objective: Help students organize the new concepts they will learn in this chapter.

Online Edition
Multilingual Glossary

Resources

Learn It Online
Multilingual Glossary

Possible answers to *Vocabulary Connections*

1. It will be a polynomial of degree 2.
2. a curve
3. a point on a graph of a curve with the greatest *y*-coordinate
4. an imaginary line that splits a figure such that one side of the figure is a mirror image of the other

CHAPTER 8 Study Guide: Preview

Where You've Been

Previously, you

- identified and graphed linear functions.
- transformed linear functions.
- solved linear equations.
- factored quadratic polynomials, including perfect-square trinomials.

In This Chapter

You will study

- identifying and graphing quadratic functions.
- transforming quadratic equations.
- solving quadratic equations.
- using factoring to graph quadratic functions and solve quadratic equations.

Where You're Going

You can use the skills in this chapter

- to determine the maximum height of a ball thrown into the air.
- to graph higher-degree polynomials in future math classes, including Algebra 2.
- to solve problems about the height of launched or thrown objects in Physics.

Key Vocabulary/Vocabulario

axis of symmetry	eje de simetría
completing the square	completar el cuadrado
maximum	máximo
minimum	mínimo
parabola	parábola
quadratic equation	ecuación cuadrática
quadratic function	función cuadrática
vertex	vértice
zero of a function	cero de una función

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The value of a function is determined by its rule. The rule is an algebraic expression. What is true about the algebraic expression that determines a **quadratic function**?
2. The shape of a **parabola** is similar to the shape of an open parachute. Predict the shape of a *parabola*.
3. A **minimum** is a point on the graph of a curve with the least *y*-coordinate. How might a **maximum** be described?
4. An axis is an imaginary line. Use this information and your understanding of symmetry to define the term **axis of symmetry**.

MATHEMATICAL PRACTICES

The Common Core Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Opportunities to develop these practices are integrated throughout this program. Some examples are provided below.

1. **Make sense of problems and persevere in solving them.** Pages 528–529, 536–537, 539, 541–542, 549–550, 552, 557–558, 565–567, 571–573, 577, 579–581, 587–589, 594–597, 598
2. **Reason abstractly and quantitatively.** Pages 527, 543, 549, 572
3. **Construct viable arguments and critique the reasoning of others.** Pages 525, 528–529, 535, 536–537, 541, 542, 548, 550, 556, 558, 565, 566, 571, 572–573, 578, 580–581, 587, 588, 593, 595–596
4. **Model with mathematics.** Pages 534, 548, 556, 564, 570, 578, 585, 593
5. **Use appropriate tools strategically.** Pages 526–529, 532, 537, 544, 549–551, 554, 557–559, 560, 574, 576, 580, 586
6. **Attend to precision.** Pages 528, 572, 595
7. **Look for and make use of structure.** Pages 543, 550, 581, 597
8. **Look for and express regularity in repeated reasoning.** Pages 522, 530, 584

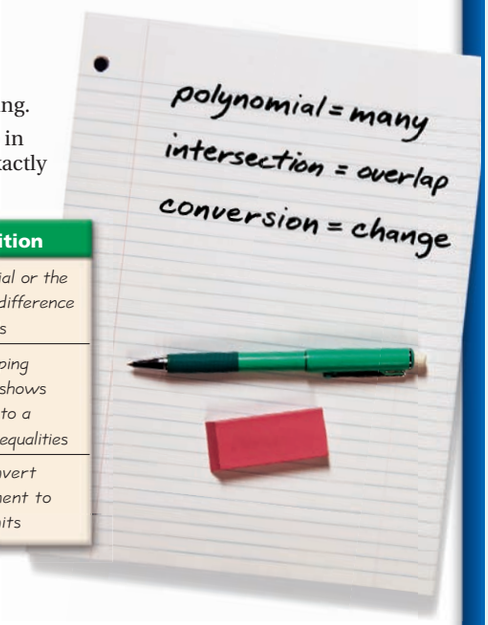
520 Chapter 8

Study Strategy: Learn Vocabulary

Mathematics has a vocabulary all its own. Many new terms appear on the pages of your textbook. Learn these new terms as they are introduced. They will give you the necessary tools to understand new concepts.

Some tips to learning new vocabulary include:

- Look at the **context** in which a new word appears.
- Use **prefixes** or **suffixes** to figure out the word's meaning.
- Relate the new term to familiar **everyday words.** Keep in mind that a word's mathematical meaning may not exactly match its everyday meaning.



Vocabulary Word	Study Tip	Definition
Polynomial	*The prefix "poly-" means many.*	*One monomial or the sum or the difference of monomials*
Intersection	*Relate it to the meaning of the "intersection of two roads".*	*The overlapping region that shows the solution to a system of inequalities*
Conversion Factor	*Relate it to the word "convert", which means change or alter.*	*Used to convert a measurement to different units*

Try This

Complete the chart.

	Vocabulary Word	Study Tips	Definition
1.	Trinomial		
2.	Independent system		
3.	Variable		

Use the context of each sentence to define the underlined word. Then relate the word to everyday words.

4. If two linear equations in a system have the same graph, the graphs are called coincident lines, or simply the same line.

5. In the formula $d = rt$, d is isolated.

Standards for Mathematical Content High School

		LESSON
REASONING WITH EQUATIONS AND INEQUALITIES Solve equations and inequalities in one variable		
CC.9-12.A.REI.4a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.	Lab 8-8, 8-8, 8-9
CC.9-12.A.REI.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial from of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	8-6, 8-7, 8-9
INTERPRETING FUNCTIONS Analyze functions using different representations		
CC.9-12.F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*	8-1, Lab 8-2, 8-2, 8-3, TLab 8-4, 8-4, 8-5, 8-10
BUILDING FUNCTIONS Build new functions from existing functions		
CC.9-12.F.BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	TLab 8-4, 8-4

Organizer

Objective: Help students apply strategies to understand and retain key concepts.

Online Edition

Study Strategy: Learn Vocabulary

ENGLISH LANGUAGE LEARNERS

When trying to determine the meaning of a new word based on the context, students should not look only at the sentence in which it appears. Remind them to consider the preceding and following sentences as well.

Extend As students work through this chapter, have them find the vocabulary words on the lesson pages and figure out their meanings based on any of the strategies shown on this page.

Answers to *Try This*

Possible answers:

1. The prefix *tri-* means "three"; a polynomial with only 3 terms.
2. Relate it to the word *independent*, which means "alone" or "by itself"; a system that has exactly one solution.

3–5. See Additional Answers.

Reading Connection

Women in Mathematics
by Lynn M. Osen

Osen tells inspiring stories of women mathematicians who overcame enormous obstacles to make major contributions in their fields.

Activity Have students read the chapter about Hypatia, who studied both quadratic equations and Diophantine equations, which allow only integer solutions. Pose this problem: Goldfish cost $3. Guppies cost $5. Joe spent $27 on goldfish and guppies. Ask students to explain why the equation $3x + 5y = 27$, which has an infinite number of solutions, has only one solution when used to solve this problem. The solution must be positive integers.

SECTION
8A Quadratic Functions

One-Minute Section Planner

Lesson	Lab Resources	Materials
Lesson 8-1 Identifying Quadratic Functions • Identify quadratic functions and determine whether they have a minimum or maximum. • Graph a quadratic function and give its domain and range. ☐ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab	**Optional** graphing calculator, index cards, sticky notes
Algebra Lab Explore the Axis of Symmetry • Find the equation of the axis of symmetry for a quadratic function. ☑ SAT-10 ☑ NAEP ☐ ACT ☑ SAT ☐ SAT Subject Tests	***Algebra Lab Activities*** Lab Recording Sheet	
Lesson 8-2 Characteristics of Quadratic Functions • Find the zeros of a quadratic function from its graph. • Find the axis of symmetry and the vertex of a parabola. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab	**Optional** graphing calculator, jump rope
Lesson 8-3 Graphing Quadratic Functions • Graph a quadratic function in the form $y = ax^2 + bx + c$. ☐ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab	**Optional** graphing calculator
Technology Lab The Family of Quadratic Functions • Use a graphing calculator to explore how the value of a affects the graph of $y = ax^2$. ☐ SAT-10 ☑ NAEP ☐ ACT ☑ SAT ☑ SAT Subject Tests	***Technology Lab Activities*** Lab Recording Sheet	**Required** graphing calculator
Lesson 8-4 Transforming Quadratic Functions • Graph and transform quadratic functions. ☐ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab ***Technology Lab Activities*** Technology Lab	**Optional** graphing calculator

Note: If NAEP is checked, the content is tested on either the Grade 8 or the Grade 12 NAEP assessment.

MK = *Manipulatives Kit*

Selected Answers in Student Edition

The answers to the following exercises are included in the Selected Answers section at the back of the Student Edition textbook.

Lesson 1: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 53, 55, 57, 59, 61b, 61c, 61d, 65, 67

Lesson 2: 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45

Lesson 3: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27b, 27c, 31, 35a, 35b, 37, 39, 41

Lesson 4: 1, 3, 5, 7, 9a, 9b, 11, 13, 19, 21, 25, 29, 31, 39, 41, 43a, 43b, 43c

All answers for the Check it Out! exercises in each lesson and for the Chapter Study Guide: Review are included in the Student Edition.

Math Background

COMMON CORE PROFESSIONAL DEVELOPMENT

QUADRATIC FUNCTIONS

Lesson 8-1

A *quadratic function* is any function that can be written in the form $y = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. This is called the *standard form* of a quadratic function.

This lesson shows several ways to identify a quadratic function. One of these is to use a table of values. Recall that linear functions have a constant rate of change. Therefore, in a table of values for a linear function, a constant change in the x-values corresponds to a constant change in the y-values. For quadratic functions, it is the rate of change itself that has a constant rate of change. In other words, when there is a constant change in x-values, a quadratic function has *constant second differences.* The table illustrates this for the quadratic function $y = x^2 + 3$.

x	2	3	4	5	6
y	7	12	19	28	39

First differences: + 5 + 7 + 9 + 11

Second differences: + 2 + 2 + 2

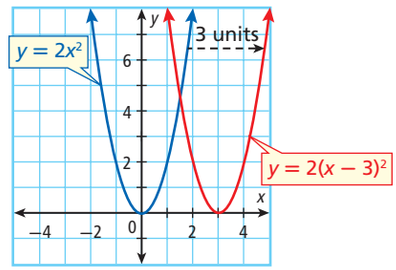
DEVELOPING QUADRATIC FUNCTIONS

LESSONS 8-1 to 8-4

The simplest quadratic function is $y = x^2$. Its graph is a *parabola* that opens upward, has its vertex at the origin, and is symmetric about the y-axis. The graphs of all other quadratic functions may be built by performing a series of transformations on the graph of $y = x^2$.

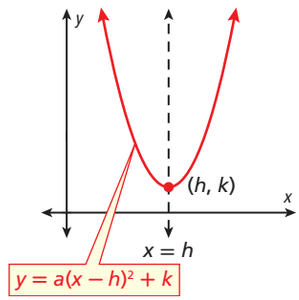
For functions in the form $y = ax^2$ ($a \neq 0$), the value of a determines the direction and shape of the parabola. If $a > 0$, the parabola opens upward; if $a < 0$, the parabola opens downward. If $|a| > 1$, the parabola is narrower than the graph of $y = x^2$; if $|a| < 1$, the parabola is wider than the graph of $y = x^2$.

The graph of a function in the form $y = a(x - h)^2$ is a horizontal translation of the graph of $y = ax^2$. For example, the graph of $y = 2(x - 3)^2$ is identical to the graph of $y = 2x^2$, but shifted 3 units to the right.



Finally, the constant k in $y = a(x - h)^2 + k$ represents a vertical translation of the graph of $y = a(x - h)^2$. The translation is k units upward if $k > 0$ and $|k|$ units downward if $k < 0$.

The equation $y = a(x - h)^2 + k$ is called the *vertex form* of a quadratic function. This form makes it easy to identify the vertex of the parabola, (h, k), and the axis of symmetry, $x = h$. (Students will study vertex form in Algebra 2.)



One way to find the formula for the axis of symmetry of a parabola $\left(x = -\frac{b}{2a}\right)$ is by using vertex form. As shown below, the basic idea is to transform vertex form into standard form and then write the equation of the axis of symmetry, $x = h$, in terms of a, b, and c.

$$y = a(x - h)^2 + k$$
$$y = a(x^2 - 2hx + h^2) + k$$
$$y = ax^2 - 2ahx + ah^2 + k$$

In standard form, the value of b is the coefficient of x, so $b = -2ah$. Solving for h gives $h = -\frac{b}{2a}$. Thus, the axis of symmetry is the vertical line $x = -\frac{b}{2a}$.

8-1 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Identify quadratic functions and determine whether they have a minimum or maximum.

Graph a quadratic function and give its domain and range.

Algebra Lab
In *Algebra Lab Activities*

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

1. Evaluate $x^2 + 5x$ for $x = 4$ and $x = -3$. 36; −6

2. Generate ordered pairs for the function $y = x^2 + 2$ with the given domain.
D: {−2, −1, 0, 1, 2}

x	−2	−1	0	1	2
y	6	3	2	3	6

Also available online

Math Humor

Q: Why can't quadratics ever be considered cool?

A: One of their terms is always a square.

COMMON CORE

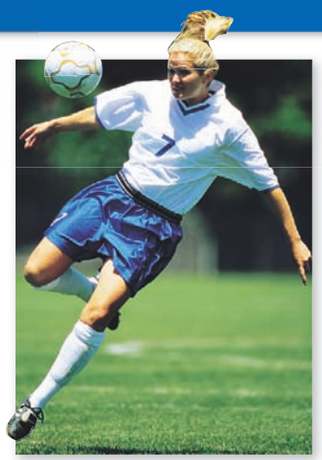
CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
CC.9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve,….

Learn It Online
State Resources Online

COMMON CORE

8-1

Identifying Quadratic Functions



CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, …. *
a. Graph … quadratic functions and show intercepts, maxima, and minima. *Also* **CC.9-12.A.REI.10**

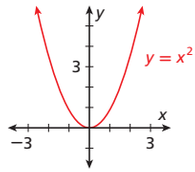
Objectives
Identify quadratic functions and determine whether they have a minimum or maximum.

Graph a quadratic function and give its domain and range.

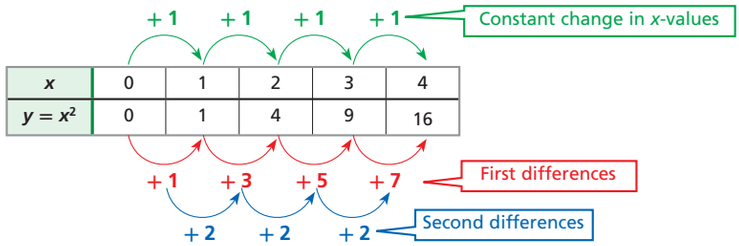
Vocabulary
quadratic function
parabola
vertex
minimum
maximum

Why learn this?
The height of a soccer ball after it is kicked into the air can be described by a quadratic function. (See Exercise 51.)

The function $y = x^2$ is shown in the graph. Notice that the graph is not linear. This function is a *quadratic function*. A **quadratic function** is any function that can be written in the standard form $y = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. The function $y = x^2$ can be written as $y = 1x^2 + 0x + 0$, where $a = 1$, $b = 0$, and $c = 0$.



Previously, you identified linear functions by finding that a constant change in x corresponded to a constant change in y. The differences between y-values for a **constant change in x-values** are called ***first differences***.



Notice that the quadratic function $y = x^2$ does not have constant first differences. It has constant ***second differences***. This is true for all quadratic functions.

EXAMPLE 1 **Identifying Quadratic Functions**

Tell whether each function is quadratic. Explain.

A

x	y
−4	8
−2	2
0	0
2	2
4	8

+2, +2, +2, +2 | −6, −2, +2, +6 | +4, +4, +4

Since you are given a table of ordered pairs with a constant change in x-values, see if the second differences are constant.

Find the first differences, then find the second differences.

The function is quadratic. The second differences are constant.

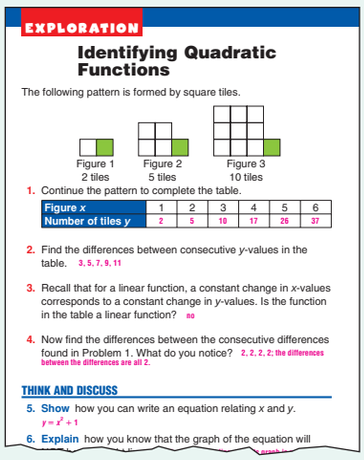
B $y = -3x + 20$ *Since you are given an equation, use $y = ax^2 + bx + c$.*

This is not a quadratic function because the value of a is 0.

Caution!
Be sure there is a constant change in x-values before you try to find first or second differences.

Afro Foto Agency

1 Introduce



EXPLORATION

Identifying Quadratic Functions

The following pattern is formed by square tiles.

Figure 1: 2 tiles | Figure 2: 5 tiles | Figure 3: 10 tiles

1. Continue the pattern to complete the table.

Figure x	1	2	3	4	5	6
Number of tiles y	2	5	10	17	26	37

2. Find the differences between consecutive y-values in the table. 3, 5, 7, 9, 11

3. Recall that for a linear function, a constant change in x-values corresponds to a constant change in y-values. Is the function in the table a linear function? no

4. Now find the differences between the consecutive differences found in Problem 1. What do you notice? 2, 2, 2, 2; the differences between the differences are all 2.

THINK AND DISCUSS

5. **Show** how you can write an equation relating x and y. $y = x^2 + 1$

6. **Explain** how you know that the graph of the equation will

Motivate

Remind students that data representing a linear relationship will have constant first differences. Then explain that data representing a quadratic relationship can also be identified from a table of values that shows constant second differences.

Explorations and answers are provided in your online resources.

Helpful Hint

In a quadratic function, only a cannot equal 0. It is okay for the values of b and c to be 0.

Tell whether each function is quadratic. Explain.

C $y + 3x^2 = -4$

$-3x^2 \quad -3x^2$

$y = -3x^2 - 4$

Try to write the function in the form $y = ax^2 + bx + c$ by solving for y. Subtract $3x^2$ from both sides.

This is a quadratic function because it can be written in the form $y = ax^2 + bx + c$ where $a = -3$, $b = 0$, and $c = -4$.

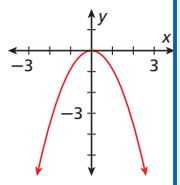
1a. Yes; the second differences are constant.

1b. Yes; the function can be written in the form $y = ax^2 + bx + c$.

CHECK IT OUT! **Tell whether each function is quadratic. Explain.**

1a. $\{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$ **1b.** $y + x = 2x^2$

The graph of a quadratic function is a curve called a **parabola**. To graph a quadratic function, generate enough ordered pairs to see the shape of the parabola. Then connect the points with a smooth curve.



EXAMPLE 2 Graphing Quadratic Functions by Using a Table of Values

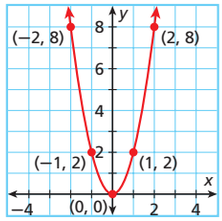
Use a table of values to graph each quadratic function.

A $y = 2x^2$

x	$y = 2x^2$
−2	8
−1	2
0	0
1	2
2	8

Make a table of values. Choose values of x and use them to find values of y.

Graph the points. Then connect the points with a smooth curve.

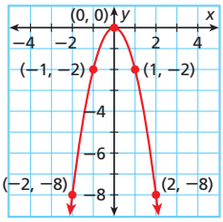


B $y = -2x^2$

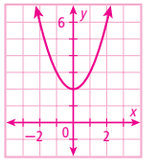
x	$y = -2x^2$
−2	−8
−1	−2
0	0
1	−2
2	−8

Make a table of values. Choose values of x and use them to find values of y.

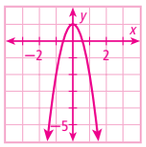
Graph the points. Then connect the points with a smooth curve.



2a.



2b.



CHECK IT OUT! **Use a table of values to graph each quadratic function.**

2a. $y = x^2 + 2$ **2b.** $y = -3x^2 + 1$

As shown in the graphs in Examples 2A and 2B, some parabolas open upward and some open downward. Notice that the only difference between the two equations is the value of a. When a quadratic function is written in the form $y = ax^2 + bx + c$, the value of a determines the direction a parabola opens.

- A parabola opens **upward** when $a > 0$.
- A parabola opens **downward** when $a < 0$.

COMMON ERROR ALERT

Students might think a function rule needs three terms to be quadratic. Remind them that the values of b and c can equal 0; only a cannot be 0.

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Additional Examples

Example 1

Tell whether each function is quadratic. Explain.

A. $\{(-2, -9), (-1, -2), (0, -1), (1, 0), (2, 7)\}$ No; the second differences are not constant.

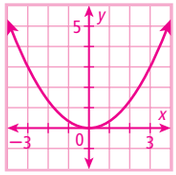
B. $y = 7x + 3$ No; there is no x^2-term, so $a = 0$.

C. $y - 10x^2 = 9$ Yes; it can be written in the form $y = ax^2 + bx + c$, where $a = 10$, $b = 0$, and $c = 9$.

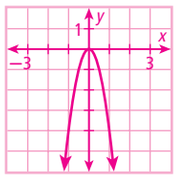
Example 2

Use a table of values to graph each quadratic function.

A. $y = \frac{1}{3}x^2$



B. $y = -4x^2$



Also available online

2 Teach

Guided Instruction

Show students how to identify quadratic functions given a table of values or a function rule. Explain that the graph of a quadratic function is a parabola. Point out the parabolic shape, the symmetrical features, and the vertex of the graph.

Explain that the value of a in $y = ax^2 + bx + c$ determines whether the function has a minimum or a maximum value. This minimum or maximum value determines the range of the function.

Reaching All Learners

Through Kinesthetic Experience

Display each function below one at a time. Have students discuss, in pairs, whether to lift their arms up in the shape of a U if the graph opens upward or down in a shape of an upside-down U if the graph opens downward. Then have students demonstrate their decision.

$y = -3x^2 + 18$	down
$y = 5x + 8 - \frac{1}{5}x^2$	down
$-2x^2 + y = -5$	up
$y - \frac{1}{2}x^2 + 10 - x = 0$	up
$3x - y = -x^2$	up

Questioning Strategies

EXAMPLE 1

- How do you find the second differences of a set of ordered pairs with a constant change in x-values?
- How can you tell by looking at an equation whether the value of a is 0?

EXAMPLE 2

- How do you know when you have enough values in the table to graph the parabola?

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Additional Examples

Example 3

Tell whether the graph of each quadratic function opens upward or downward. Explain.

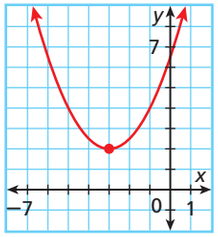
A. $y - \frac{1}{4}x^2 = x - 3$ upward; $a > 0$

B. $y = 5x - 3x^2$ downward; $a < 0$

Example 4

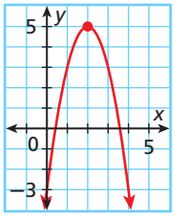
Identify the vertex of each parabola. Then give the minimum or maximum value of the function.

A.



vertex: $(-3, 2)$; minimum: 2

B.



vertex: $(2, 5)$; maximum: 5

Also available online

Mathematical Practices Questioning Strategies

EXAMPLE 3

- How do you determine from an equation of a quadratic function whether the graph will open upward or downward?
- Why is it necessary that the function be in the form $y = ax^2 + bx + c$ before determining whether the graph opens upward or downward?

EXAMPLE 4

- Are minimums and maximums x-values or y-values?

EXAMPLE 3 Identifying the Direction of a Parabola

Tell whether the graph of each quadratic function opens upward or downward. Explain.

A $y = 4x^2$

$y = 4x^2$

$a = 4$ *Identify the value of a.*

Since $a > 0$, the parabola opens **upward**.

B $2x^2 + y = 5$

$2x^2 + y = 5$

$-2x^2 \quad -2x^2$ *Write the function in the form $y = ax^2 + bx + c$ by solving for y. Subtract $2x^2$ from both sides.*

$y = -2x^2 + 5$

$a = -2$ *Identify the value of a.*

Since $a < 0$, the parabola opens **downward**.

CHECK IT OUT! **Tell whether the graph of each quadratic function opens upward or downward. Explain.**

3a. $f(x) = -4x^2 - x + 1$ **3b.** $y - 5x^2 = 2x - 6$

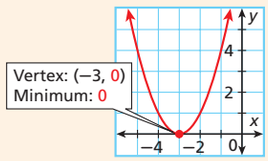
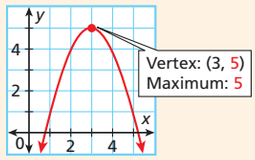
3a. Because $a < 0$, the parabola opens downward.

3b. Because $a > 0$, the parabola opens upward.

The highest or lowest point on a parabola is the **vertex**. If a parabola opens upward, the vertex is the lowest point. If a parabola opens downward, the vertex is the highest point.

Know it! Note

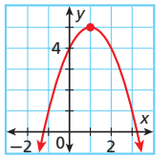
Minimum and Maximum Values

WORDS	If $a > 0$, the parabola opens upward, and the y-value of the vertex is the **minimum** value of the function.	If $a < 0$, the parabola opens downward, and the y-value of the vertex is the **maximum** value of the function.
GRAPHS	$y = x^2 + 6x + 9$ 	$y = -x^2 + 6x - 4$ 

EXAMPLE 4 Identifying the Vertex and the Minimum or Maximum

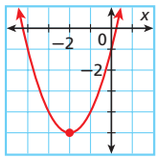
Identify the vertex of each parabola. Then give the minimum or maximum value of the function.

A



The vertex is $(1, 5)$, and the maximum is 5.

B



The vertex is $(-2, -5)$, and the minimum is -5.

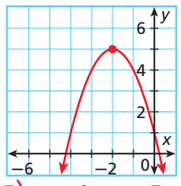
Teaching Tip **Multiple Representations** Have students graph several quadratic functions using a graphing calculator, and make a conjecture about whether a graph opens upward or downward prior to viewing the graph.

Teaching Tip **Reading Math** Discuss with students the meanings of *minimum* and *maximum* in common usage. Connect these everyday meanings with those that are used with parabolas by associating them with the maximum value or minimum value of a quadratic function.

ENGLISH LANGUAGE LEARNERS

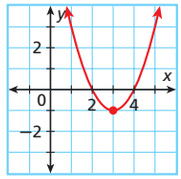
CHECK IT OUT! Identify the vertex of each parabola. Then give the minimum or maximum value of the function.

4a.



vertex: $(-2, 5)$; maximum: 5

4b.

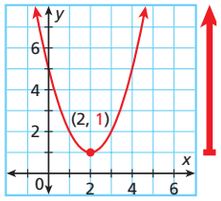


vertex: $(3, -1)$; minimum: -1

Caution!

You may not be able to see the entire graph, but that does not mean the graph stops. Remember that the arrows indicate that the graph continues.

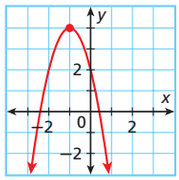
Unless a specific domain is given, you may assume that the domain of a quadratic function is all real numbers. You can find the range of a quadratic function by looking at its graph.



For the graph of $y = x^2 - 4x + 5$, the **range** begins at the minimum value of the function, where $y = 1$. All the y-values of the function are greater than or equal to 1. So the range is $y \geq 1$.

EXAMPLE 5 Finding Domain and Range

Find the domain and range.

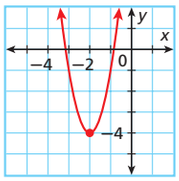


Step 1 The graph opens downward, so identify the maximum.
The vertex is $(-1, 4)$, so the maximum is 4.

Step 2 Find the domain and range.
D: all real numbers
R: $y \leq 4$

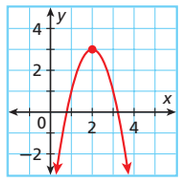
CHECK IT OUT! Find the domain and range.

5a.



5a. D: all real numbers; R: $y \geq -4$

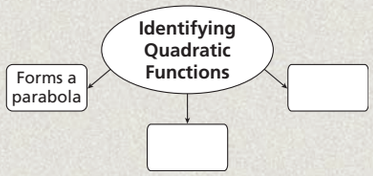
5b.



5b. D: all real numbers; R: $y \leq 3$

THINK AND DISCUSS

1. How can you identify a quadratic function from ordered pairs? from looking at the function rule?
2. **GET ORGANIZED** Copy and complete the graphic organizer below. In each box, describe a way of identifying quadratic functions.



Know it! Note

COMMON ERROR ALERT

Some students may automatically say that the function has a minimum when a parabola opens downward, and maximum when a parabola opens upward because of the word association. Tell students to visualize the graph before determining whether it has a minimum or maximum.

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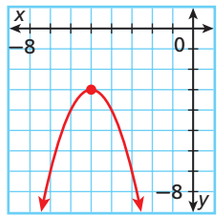
Additional Examples

Example 5

Find the domain and range.



D: all real numbers
R: $y \leq -3$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 5

- Is the set of all real numbers the domain or the range? Is this true for all quadratic functions?

3 Close

Summarize

Write $y = 3x^2$ on the board. Ask students for the values of a, b, and c. Then have students list five ordered pairs for this function and find the second differences. Ask whether the graph would open upward or downward and whether the parabola would have a minimum or maximum. Graph the function by plotting the ordered pairs. Continue with other quadratic equations as time allows.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. If the second differences are constant, the function is quadratic. If the function can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$, it is quadratic.
2. See Additional Answers.

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–3**

Basic	22–32, 45–50, 53–60
Average	22–32, 45–50, 53–60
Advanced	22–32, 45–50, 53–60, 68

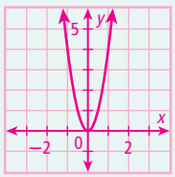
If you finished Examples **1–5**

Basic	22–50, 53–60, 61–67
Average	22–69
Advanced	22–69

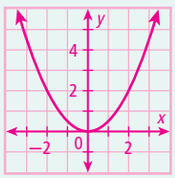
Homework Quick Check
Quickly check key concepts.
Exercises: 24, 26, 30, 34, 38

Answers

6.



7.



MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 52, 61, 63, 68

Reason abstractly and quantitatively. Exercises 39–44

Construct viable arguments and critique the reasoning of others. Exercises 62, 64

Use appropriate tools strategically. Exercises 6–9, 26–29, 69

Attend to precision. Exercise 51

Learn It Online
State Resources Online

8-1 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. **Vocabulary** The y-value of the vertex of a parabola that opens upward is the ____?____ value of the function. (*maximum* or *minimum*) **minimum**

SEE EXAMPLE 1 Tell whether each function is quadratic. Explain.

2. $y + 6x = -14$ **2. No; $a = 0$**
3. $2x^2 + y = 3x - 1$ **3. Yes; the function can be written in the form $y = ax^2 + bx + c$.**
4.

x	−4	−3	−2	−1	0
y	39	18	3	−6	−9

Yes; the second differences are constant.

5. $\{(-10, 15), (-9, 17), (-8, 19), (-7, 21), (-6, 23)\}$ **No; the first differences are constant, so the function is linear.**

SEE EXAMPLE 2 Use a table of values to graph each quadratic function.

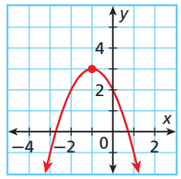
6. $y = 4x^2$
7. $y = \frac{1}{2}x^2$
8. $y = -x^2 + 1$
9. $y = -5x^2$

SEE EXAMPLE 3 Tell whether the graph of each quadratic function opens upward or downward. Explain.

10. $y = -3x^2 + 4x$ **10. downward; $a < 0$**
11. $y = 1 - 2x + 6x^2$ **11. $a > 0$; upward**
12. $y + x^2 = -x - 2$ **12. downward; $a < 0$**
13. $y + 2 = x^2$ **13. upward; $a > 0$**
14. $y - 2x^2 = -3$ **14. upward; $a > 0$**
15. $y + 2 + 3x^2 = 1$ **15. downward; $a < 0$**

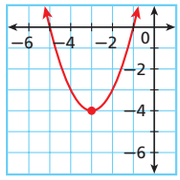
SEE EXAMPLE 4 Identify the vertex of each parabola. Then give the minimum or maximum value of the function.

16.



(−1, 3); maximum: 3

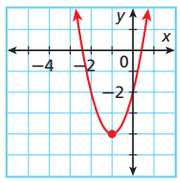
17.



(−3, −4); minimum: −4

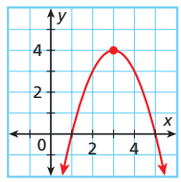
SEE EXAMPLE 5 Find the domain and range.

18.



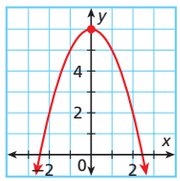
D: all real numbers; R: $y \geq -4$

19.



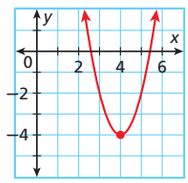
D: all real numbers; R: $y \leq 4$

20.



D: all real numbers; R: $y \leq 6$

21.



D: all real numbers; R: $y \geq -4$

READING STRATEGIES

Study the information about the example below to better understand quadratic functions.

Quadratic functions are written in the form $y = ax^2 + bx + c$, where $a \neq 0$.

$y = -\frac{1}{2}x^2 + 3$

$a = -\frac{1}{2}$, $b = 0$ and $c = 3$

The graph of a quadratic function is a *parabola*.

vertex: (0, 3); *maximum*: 3

Recall that the domain is the set of x-values and the range is the set of y-values for a function.

You can create a table of values to graph a quadratic function.

x	y
−4	−5
−2	1
0	3
2	1
4	−5

The graph of $y = -\frac{1}{2}x^2 + 3$ opens downward because $a < 0$.

D: all real numbers
R: $y \leq 3$

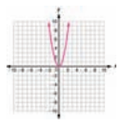
Complete the following.

1. Identify the values of a, b, and c in the quadratic function $y = 3x^2 - 5x + 2$.
   $a = 3$, $b = -5$, $c = 2$
2. Does the graph of the function $y = 3x^2 - 5x + 2$ open upward or downward?
   upward
3. Identify the domain and range for a function whose graph opens upward with vertex (0, −5).
   D: all real numbers; R: $y \geq -5$

Answer the following about the function $y = 2x^2$.

4. Create a table of values for the function.
5. Graph the function.
6. What is the vertex of the graph?
   (0, 0)
7. Does the graph open upward or downward? Why?
   upward; because $a > 0$.

x	y
−2	8
−1	2
0	0
1	2
2	8



RETEACH

There are three steps to identify a quadratic function from a table of ordered pairs.

Tell whether this function is quadratic. Explain.

x	y
−5	−191
−3	−59
−1	1
1	−11
3	−95

$-3 - (-5) = +2$, $-1 - (-3) = +2$, $1 - (-1) = +2$, $3 - 1 = +2$

$-59 - (-191) = +132$, $1 - (-59) = +60$, $-11 - 1 = -12$, $-95 - (-11) = -84$

$60 - 132 = -72$, $-12 - 60 = -72$, $-84 - (-12) = -72$

Step 1: Check for a constant change in x-values. Calculate the second value minus the first.

Step 2: Find the first differences in y-values. If they are constant, the function is linear.

Step 3: Find the second differences in y-values. If the second differences are constant, then the function is quadratic.

This function is quadratic because the second differences are constant.

Tell whether each function is quadratic. Explain.

1.

x	y
−4	43
−1	16
2	7
5	16
8	43

$-1 - (-4) = +3$, $2 - (-1) = +3$, $5 - 2 = +3$, $8 - 5 = +3$

$16 - 43 = -27$, $7 - 16 = -9$, $16 - 7 = +9$, $43 - 16 = +27$

$-9 - (-27) = +18$, $9 - (-9) = +18$, $27 - (9) = +18$

quadratic; the second difference are constant.

2. 3.

x	y
−2	12
−1	4
0	0
1	6
2	28

+1, +1, +1, +1; −8, −4, +6, +22; +4, +10, +16

not quadratic the second differences are not constant.

x	y
−6	−18
−4	−14
−2	−10
0	−6
2	−2

not quadratic it is linear.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
22–25	1
26–29	2
30–32	3
33–34	4
35–38	5

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

22. No; the second differences are not constant.

Tell whether each function is quadratic. Explain.

22.

x	−2	−1	0	1	2
y	−1	0	4	9	15

23. $-3x^2 + x = y - 11$ Yes; the function can be written in the form $y = ax^2 + bx + c$.

24. $\{(0, -3), (1, -2), (2, 1), (3, 6), (4, 13)\}$ Yes; the second differences are constant.

25. $y = \frac{2}{3}x - \frac{4}{9} + \frac{1}{6}x^2$ Yes; the function can be written in the form $y = ax^2 + bx + c$.

Use a table of values to graph each quadratic function.

26. $y = x^2 - 5$ **27.** $y = -\frac{1}{2}x^2$ **28.** $y = -2x^2 + 2$ **29.** $y = 3x^2 - 2$

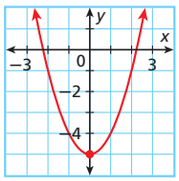
Tell whether the graph of each quadratic function opens upward or downward. Explain.

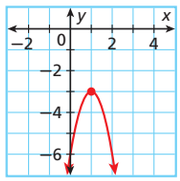
30. $y = 7x^2 - 4x$ upward; $a > 0$

31. $x - 3x^2 + y = 5$ upward; $a > 0$

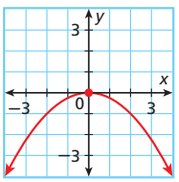
32. $y = -\frac{2}{3}x^2$ downward; $a < 0$

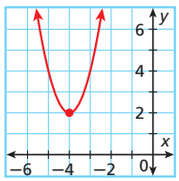
Identify the vertex of each parabola. Then give the minimum or maximum value of the function.

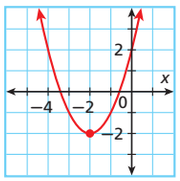
33.  vertex: $(0, -5)$; minimum: -5

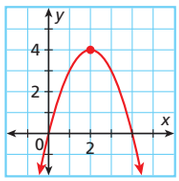
34.  vertex: $(1, -3)$; maximum: -3

Find the domain and range.

35.  D: all real numbers; R: $y \le 0$

36.  D: all real numbers; R: $y \ge 2$

37.  D: all real numbers; R: $y \ge -2$

38.  D: all real numbers; R: $y \le 4$

Tell whether each statement is sometimes, always, or never true.

39. The graph of a quadratic function is a straight line. never

40. The range of a quadratic function is the set of all real numbers. never

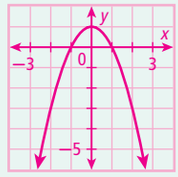
41. The highest power of the independent variable in a quadratic function is 2. always

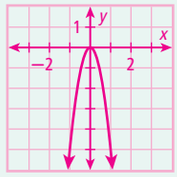
42. The graph of a quadratic function contains the point $(0, 0)$. sometimes

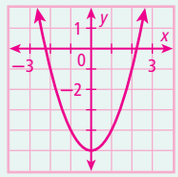
43. The vertex of a parabola occurs at the minimum value of the function. sometimes

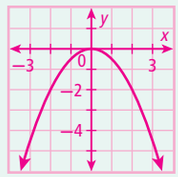
44. The graph of a quadratic function that has a minimum opens upward. always

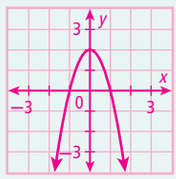
Answers

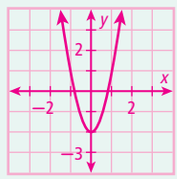
8. 

9. 

26. 

27. 

28. 

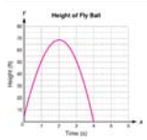
29. 

PROBLEM SOLVING

Write the correct answer.

1. During a softball game, Kay hit a fly ball. The function $f(x) = -16t^2 + 64t + 4$ describes the height of the softball in feet. Make a table of values for the function and then graph it.

x	0	1	2	3	4
$f(x)$	4	52	68	52	4



2. Jorge recorded the number of customers y that came to his store over a number of hours x. Does the data represent a quadratic function? Explain.

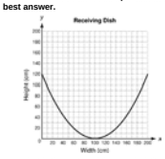
x	1	3	5	7	9
y	2	5	11	17	23

No; the second differences are not constant.

3. NASA has a plane that travels in a parabola to simulate zero-gravity. Its path can be modeled by the equation $y = -8.6x^2 + 560x + 24000$ where y is the altitude in feet and x is the time since it started the maneuver. What is a reasonable domain for this function?

$x \ge 0$

Radio telescopes are built in the shape of a parabola. The graph below shows a radio telescope dish in cross-section. Select the best answer.



4. What is the vertex of this parabola?

A (0, 120) C (200, 120)
B (100, 0) D (100, 120)

5. What are the domain and range of this function?

F D: all real numbers
R: all real numbers
G D: $x \ge 0$, R: $y \ge 0$
H D: $x \le 200$ R: $y \le 120$
J D: $0 \le x \le 200$ R: $0 \le y \le 120$

6. Which of the following could be the equation used by engineers to construct the radio telescope dish?

A $y = 1.2x + 120$
B $y = -1.2x + 120$
C $y = 0.012x^2 - 2.4x + 120$
D $y = -0.012x^2 - 2.4x + 120$

CHALLENGE

In Lesson 9–1 you saw that quadratic functions have constant ***second differences*** between y-values when there is a constant change between the x-values. You can use the form $y = ax^2 + bx + c$ to prove this.

Use this table to organize your work as you follow the steps provided below.

x	y	first differences	second differences
n	$an^2 + bn + c$		
$n + 1$	$a(n+1)^2 + b(n+1) + c =$ $a(n^2 + 2n + 1) + b(n+1) + c =$ $an^2 + 2an + a + bn + b + c$	$an^2 + 2an + a + bn + b + c$ $-(an^2 + bn + c)$ $2an + a + b$	
$n + 2$	$a(n+2)^2 + b(n+2) + c =$ $a(n^2 + 4n + 4) + b(n+2) + c =$ $an^2 + 4an + 4a + bn + 2b + c$	$an^2 + 4an + 4a + bn + 2b + c$ $-(an^2 + 2an + a + bn + b + c)$ $2an + 3a + b$	$2an + 3a + b$ $-(2an + a + b)$ $2a$
$n + 3$	$a(n+3)^2 + b(n+3) + c =$ $a(n^2 + 6n + 9) + b(n+3) + c =$ $an^2 + 6an + 9a + bn + 3b + c$	$an^2 + 6an + 9a + bn + 3b + c$ $-(an^2 + 4an + 4a + bn + 2b + c)$ $2an + 5a + b$	$2an + 5a + b$ $-(2an + 3a + b)$ $2a$

1. You want to prove that the second differences between y-values are constant regardless of the x-values, so let the variable n represent the first x-value, and let the constant change between x-values be 1. Write the next three x-values in the table.
2. In the y-column of the table, substitute each x-value into the expression $ax^2 + bx + c$. Multiply and combine like terms to simplify each polynomial.
3. Subtract polynomials to find the first differences.
4. Are the first differences constant? Explain.

No, the middle terms (a, $3a$, $5a$) are different.

5. Subtract polynomials to find the second differences.
6. Are the second differences constant? Explain.

Yes, the second differences are $2a$.

7. In general, for any quadratic function of the form $y = ax^2 + bx + c$, when the change between x-values is 1, what will be the second differences between y-values? $2a$
8. If you make a table of coordinates for these quadratic functions, and use a constant change of 1 between x-values, what will be the second differences? (Don't actually make a table of values!)
   a. $y = 5x^2 + 7x - 8$ $2(5) = 10$
   b. $y = -3x^2 + 12$ $2(-3) = -6$

PRACTICE A
PRACTICE C
PRACTICE B

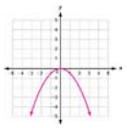
Tell whether each function is quadratic. Explain.

1. (0, 6), (1, 12), (2, 20), (3, 30) yes; the second differences are constant.
2. $3x + 2y = 8$ no; it cannot be written in the form $y = ax^2 + bx + c$.

Use a table of values to graph each quadratic function.

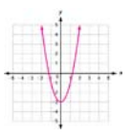
3. $y = -\frac{1}{2}x^2$

x	y
−2	−2
−1	$-\frac{1}{2}$
0	0
1	$-\frac{1}{2}$
2	−2



4. $y = 2x^2 - 3$

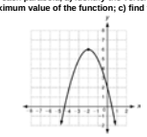
x	y
−2	5
−1	−1
0	−3
1	−1
2	5

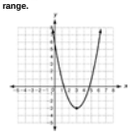


Tell whether the graph of each quadratic function opens upward or downward. Explain.

5. $y = -3x^2 + 5$ downward, $a = -3$, $a < 0$
6. $-x^2 + y = 8$ upward, $a = 1$, $a > 0$

For each parabola, a) identify the vertex; b) give the minimum or maximum value of the function; c) find the domain and range.

7. 
   a. $(-2, 6)$
   b. maximum: 6
   c. D: all real numbers; R: $y \le 6$

8. 
   a. $(3, -3)$
   b. minimum: -3
   c. D: all real numbers; R: $y \ge -3$

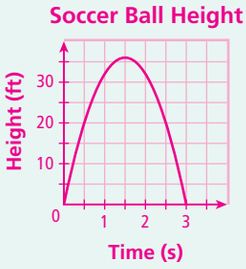
Teaching Tip **Inclusion** In **Exercise 47,** remind students that the quantity $(x + 1)^2$ means $(x + 1)(x + 1)$, not $x^2 + 1$.

Multi-Step Test Prep **Exercise 63** involves showing that the height of a rocket over time can be represented by a quadratic function. This exercise prepares students for the Multi-Step Test Prep.

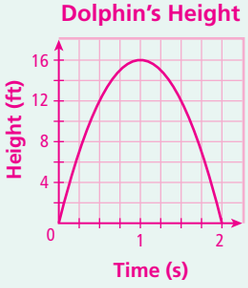
Answers

51c. The independent variable x represents the time since the volleyball is served, so this makes sense only for nonnegative numbers.

52a.



61a.



62a. For a linear function, the first differences will be constant for a constant change in x. For a quadratic function, the second differences will be constant for a constant change in x.

b. A linear function can be written in the form $y = mx + b$. A quadratic function can be written in the form $y = ax^2 + bx + c$, where a, b, and c are real numbers and a is not equal to 0.

c. The graph of a linear function is a line. The graph of a quadratic function is a parabola.

45. No; the value of a is 0.

46. yes; $y = 2x^2 + 3x - 5$

47. yes; $y = x^2 + 2x + 1$

48. yes; $y = -x^2 + 2x + 4$

49. yes; $y = 3x^2 + 0x - 9$

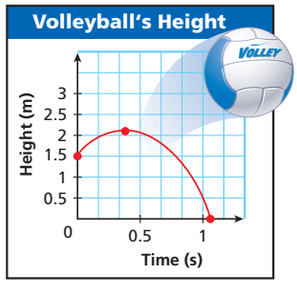
50. No; the function has a power of x greater than 2.

Tell whether each function is quadratic. If it is, write the function in standard form. If not, explain why not.

45. $y = 3x - 1$ **46.** $y = 2x^2 - 5 + 3x$ **47.** $y = (x + 1)^2$

48. $y = 5 - (x - 1)^2$ **49.** $y = 3x^2 - 9$ **50.** $y = (x + 1)^3 - x^2$

51. Estimation The graph shows the approximate height y in meters of a volleyball x seconds after it is served.



a. Estimate the time it takes for the volleyball to reach its greatest height. about 0.375 s

b. Estimate the greatest height that the volleyball reaches. about 2.25 m

c. Critical Thinking If the domain of a quadratic function is all real numbers, why is the domain of this function limited to nonnegative numbers?

52. Sports The height in feet of a soccer ball x seconds after it is kicked into the air is modeled by the function $y = 48x - 16x^2$.

a. Graph the function.

b. In this situation, what values make sense for the domain? $x \geq 0$

c. Does the soccer ball ever reach a height of 50 ft? How do you know?
No; the maximum y-value on the parabola is less than 50.

Tell whether each function is linear, quadratic, or neither.

53. $y = \frac{1}{2}x - x^2$ quadratic **54.** $y = \frac{1}{2}x - 3$ linear **55.** $y + 3 = -x^2$ quadratic **56.** $y - 2x^2 = 0$ quadratic

57. $y = \frac{1}{2}x(x^2)$ neither **58.** $y = \frac{3}{x^2}$ neither **59.** $y = \frac{3}{2}x$ linear **60.** $x^2 + 2x + 1 = y$ quadratic

61. Marine Biology A scientist records the motion of a dolphin as it jumps from the water. The function $h(t) = -16t^2 + 32t$ models the dolphin's height in feet above the water after t seconds.

a. Graph the function.

b. What domain makes sense for this situation? $t \geq 0$

c. What is the dolphin's maximum height above the water? 16 ft

d. How long is the dolphin out of the water? 2 s

62. Write About It Explain how to tell the difference between a linear function and a quadratic function when given each of the following:

a. ordered pairs **b.** the function rule **c.** the graph

Multi-Step Test Prep

63. A rocket team is using simulation software to create and study water bottle rockets. The team begins by simulating the launch of a rocket without a parachute. The table gives data for one rocket design.

Time (s)	Height (m)
0	0
1	34.3
2	58.8
3	73.5
4	78.4
5	73.5
6	58.8
7	34.3
8	0

a. Show that the data represent a quadratic function.

b. Graph the function.

c. The acceleration due to gravity is 9.8 m/s^2. How is this number related to the data for this water bottle rocket?

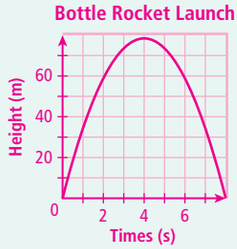
Sam Dudgeon/HMH

63a.

Time (s)	Height (m)		
0	0		
1	34.3	+ 34.3	
2	58.8	+ 24.5	− 9.8
3	73.5	+ 14.7	− 9.8
4	78.4	+ 4.9	− 9.8
5	73.5	− 4.9	− 9.8
6	58.8	− 14.7	− 9.8
7	34.3	− 24.5	− 9.8
8	0	− 34.3	

The second differences are constant.

b.



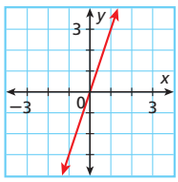
c. The acceleration is the absolute value of the second differences.

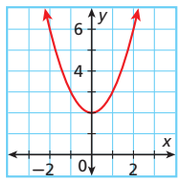
64. Critical Thinking Given the function $-3 - y = x^2 + x$, why is it incorrect to state that the parabola opens upward and has a minimum?

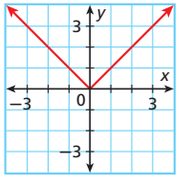
After the equation is solved for y, x^2 will be negative. This means the parabola opens downward and has a maximum.

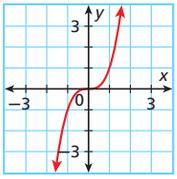
TEST PREP

65. Which of the following is the graph of a quadratic function?

Ⓐ 

Ⓒ 

Ⓑ 

Ⓓ 

66. Which of the following quadratic functions has a maximum?

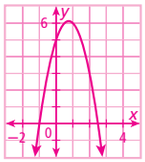
Ⓕ $2x^2 - y = 3x - 2$

Ⓖ $y = x^2 + 4x + 16$

Ⓗ $y - x^2 + 6 = 9x$

Ⓙ $y + 3x^2 = 9$

67.



67. Short Response Is the function $f(x) = 5 - 2x^2 + 3x$ quadratic? Explain your answer by using two different methods of identification. **Yes; the function can be written in the form $y = ax^2 + bx + c$, and the graph forms a parabola.**

CHALLENGE AND EXTEND

68. Multi-Step A rectangular picture measuring 6 in. by 10 in. is surrounded by a frame with uniform width x. Write a quadratic function to show the combined area of the picture and frame. **$f(x) = 4x^2 + 32x + 60$**

69. Graphing Calculator Use a graphing calculator to find the domain and range of the quadratic functions $y = x^2 - 4$ and $y = -(x + 2)^2$. **$f(x) = x^2 - 4 \rightarrow$ D: all real numbers; R: $y \geq -4$; $f(x) = -(x + 2)^2 \rightarrow$ D: all real numbers; R: $y \leq 0$**

TEST PREP DOCTOR If students chose **H** in **Exercise 66,** they may not have written the equation in the form $y = ax^2 + bx + c$. Students who chose **F** may not have divided both sides by -1 when solving for y.

Journal

Have students describe relationships between a minimum or maximum, the vertex, and the range of a quadratic function.

ALTERNATIVE ASSESSMENT

Have students choose two of the quadratic functions from **Exercises 53–60,** write each in the form $y = ax^2 + bx + c$, and identify a, b, and c. Also have them identify the vertex, the minimum or maximum, the domain, and the range for each function.

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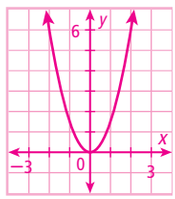
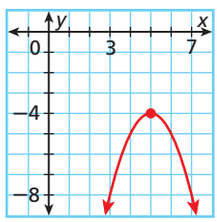
8-1 Lesson Quiz

1. Is $y = -x - 1$ quadratic? Explain. No; there is no x^2-term, so $a = 0$.

2. Use a table of values to graph $y = 1.5x^2$.



Use the graph for Problems 3–5.



3. Identify the vertex. (5, −4)

4. Does the function have a minimum or maximum? What is it? max.; −4

5. Find the domain and range. D: all real numbers; R: $y \leq -4$

Also available online

Organizer

Use with the lesson *Characteristics of Quadratic Functions*

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Find the equation of the axis of symmetry for the graph of a quadratic function.

Online Edition

Countdown

Resources

Algebra Lab Activities
Lab Recording Sheet

Teach

Discuss

When there is no x-term in a quadratic function, then $b = 0$. Because 0 divided by any nonzero number is 0, the equation of the axis of symmetry for the graph of this type of function is always $x = 0$, which is the y-axis. When there is an x-term, the axis of symmetry is *not* $x = 0$.

Close

Key Concept

The equation of the axis of symmetry for the graph of a quadratic function of the form $y = ax^2 + bx + c$ is $x = -\frac{b}{2a}$.

Assessment

Journal Have students explain how to find the equation of the axis of symmetry for the graph of a quadratic function.

MATHEMATICAL PRACTICES

Look for and express regularity in repeated reasoning.

COMMON CORE

CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* *Also* **CC.9-12.F.IF.7a**

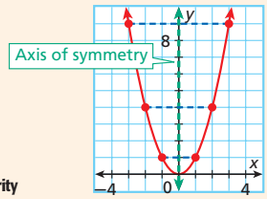
Learn It Online
State Resources Online

8-2 Algebra LAB

Use with Characteristics of Quadratic Functions

Explore the Axis of Symmetry

Every graph of a quadratic function is a parabola that is symmetric about a vertical line through its vertex called the *axis of symmetry*.



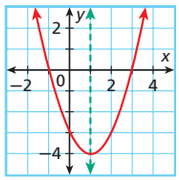
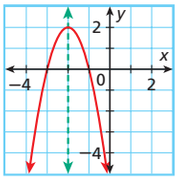
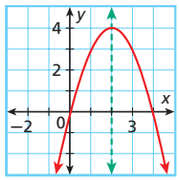
There is a relationship between a and b in the quadratic function and the equation of the axis of symmetry.

MATHEMATICAL PRACTICES **Look for and express regularity in repeated reasoning.**

CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, …. * a. Graph … quadratic functions and show intercepts, maxima, and minima.

Activity

1 Copy and complete the table.

Function	$y = 1x^2 - 2x - 3$	$y = -2x^2 - 8x - 6$	$y = -1x^2 + 4x$
Graph			
a	1	−2	−1
b	−2	−8	4
$\frac{b}{a}$	−2	4	−4
Axis of Symmetry (from graph)	$x = 1$	$x = -2$	$x = 2$

2 Compare the axis of symmetry with $\frac{b}{a}$ in your chart. What can you multiply $\frac{b}{a}$ by to get the number in the equation of the axis of symmetry? (*Hint:* Write and solve an equation to find the value.) Check your answer for each function. $-\frac{1}{2}$

3 Use your answer from Problem 2 to complete the equation of the axis of symmetry of a quadratic function. $x =$ ___?___ $x = -\frac{1}{2}\left(\frac{b}{a}\right) = \frac{-b}{2a}$

Try This

For the graph of each quadratic function, find the equation of the axis of symmetry.

1. $y = 2x^2 + 12x - 7$ $x = -3$
2. $y = 4x^2 + 8x - 12$ $x = -1$
3. $y = 5x^2 - 20x + 10$ $x = 2$
4. $y = -3x^2 + 9x + 1$ $x = \frac{3}{2}$
5. $y = x^2 - 7$ $x = 0$
6. $y = 3x^2 + x + 4$ $x = -\frac{1}{6}$

COMMON CORE

8-2 Characteristics of Quadratic Functions

CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, ….*
a. Graph … quadratic functions and show intercepts, maxima, and minima. *Also* **CC.9-12.F.IF.8, CC.9-12.F.IF.4***

Objectives
Find the zeros of a quadratic function from its graph.

Find the axis of symmetry and the vertex of a parabola.

Vocabulary
zero of a function
axis of symmetry

Who uses this?
Engineers can use characteristics of quadratic functions to find the height of the arch supports of bridges. (See Example 5.)

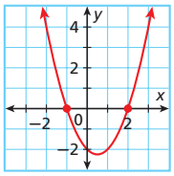
Recall that an x-intercept of a function is a value of x when $y = 0$. A **zero of a function** is an x-value that makes the function equal to 0. So a zero of a function is the same as an x-intercept of a function. Since a graph intersects the x-axis at the point or points containing an x-intercept these intersections are also at the zeros of the function. A quadratic function may have one, two, or no zeros.

Helpful Hint
Notice that if a parabola has only one zero, the zero is the x-coordinate of the vertex.

EXAMPLE 1 Finding Zeros of Quadratic Functions From Graphs

Find the zeros of each quadratic function from its graph. Check your answer.

A $y = x^2 - x - 2$

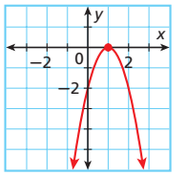


The zeros appear to be −1 and 2.

Check

$y = x^2 - x - 2$
$y = (-1)^2 - (-1) - 2$
$= 1 + 1 - 2 = 0$ ✓
$y = 2^2 - 2 - 2$
$= 4 - 2 - 2 = 0$ ✓

B $y = -2x^2 + 4x - 2$

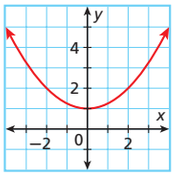


The only zero appears to be 1.

Check

$y = -2x^2 + 4x - 2$
$y = -2(1)^2 + 4(1) - 2$
$= -2(1) + 4 - 2$
$= -2 + 4 - 2$
$= 0$ ✓

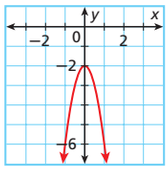
C $y = \frac{1}{4}x^2 + 1$



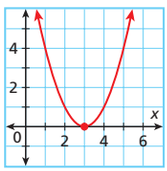
The graph does not cross the x-axis, so there are no zeros of this function.

CHECK IT OUT! **Find the zeros of each quadratic function from its graph. Check your answer.**

1a. $y = -4x^2 - 2$ no zeros



1b. $y = x^2 - 6x + 9$ 3



Gary Crabbe/Alamy

8-2 Organizer

Pacing: Traditional $1\frac{1}{2}$ days
Block $\frac{3}{4}$ day

Objectives: Find the zeros of a quadratic function from its graph.

Find the axis of symmetry and the vertex of a parabola.

Algebra Lab
In *Algebra Lab Activities*

Online Edition
Tutorial Videos

Countdown

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Warm Up

Find the x-intercept of each linear function.

1. $y = 2x - 3$ $\frac{3}{2}$
2. $y = -\frac{2}{3}x + \frac{7}{3}$ $\frac{7}{2}$
3. $y = 3x + 6$ -2

Evaluate each quadratic function for the given input values.

4. $y = -3x^2 + x - 2$, when $x = 2$ -12
5. $y = x^2 + 2x + 3$, when $x = -1$ 2

Also available online

Math Humor

Q: What do functions use to take their babies out for a walk?

A: A parabolator! (A *perambulator* is a baby carriage.)

COMMON CORE

CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
Also **CC.9-12.F.IF.8, CC.9-12.F.IF.4***

Learn It Online
State Resources Online

1 Introduce

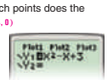
EXPLORATION

Characteristics of Quadratic Functions

In this Exploration, you will use a graphing calculator to investigate quadratic functions.

1. Enter the function $y = x^2 - 2x - 8$ as **Y1**.
2. Press 2nd TABLE to view a table of values for the function. Use the arrow keys to scroll up and down the table. For which x-values does $y = 0$? −2 and 4

3. Press GRAPH to graph the function. At which points does the graph intersect the x-axis? (−2, 0) and (4, 0)
4. Now enter the function $y = x^2 - x + 3$ as **Y1**.



5. View a table for this function. For which x-values does $y = 0$? There are no x-values for which $y = 0$.
6. View a graph of the function. At which points does the graph intersect the x-axis? There are no points at which the graph intersects the x-axis.

THINK AND DISCUSS

7. **Discuss** what you could conclude about the graph of a quadratic function that has only one value of x for which $y = 0$. The graph intersects the x-axis at only 1 point.

Motivate

Present students with some shapes, such as those below, and discuss lines of symmetry. Ask which shapes have a vertical line of symmetry and which have a horizontal line of symmetry.

Vertical: face, triangle, trapezoid, pentagon
Horizontal: arch

Explorations and answers are provided in your online resources.

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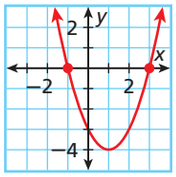
Additional Examples

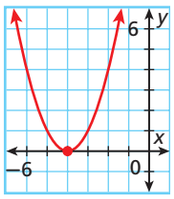
Example 1

Find the zeros of each quadratic function from its graph. Check your answer.

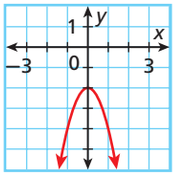
A. $y = x^2 - 2x - 3$ $-1, 3$



B. $y = x^2 + 8x + 16$ -4



C. $y = -2x^2 - 2$ no zeros



Also available online

Questioning Strategies

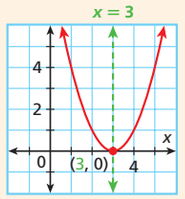
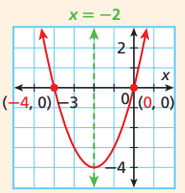
EXAMPLE 1

- Which intercepts do you look for when looking for the zeros of a function?
- How are the zeros of a quadratic function related to the vertex of its graph?

A vertical line that divides a parabola into two symmetrical halves is the **axis of symmetry**. The axis of symmetry always passes through the vertex of the parabola. You can use the zeros to find the axis of symmetry.

Know it! Note

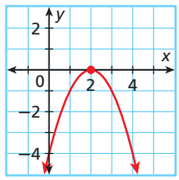
Finding the Axis of Symmetry by Using Zeros

WORDS	NUMBERS	GRAPH
One Zero		
If a function has one zero, use the x-coordinate of the vertex to find the axis of symmetry.	Vertex: $(3, 0)$ Axis of symmetry: $x = 3$	
Two Zeros		
If a function has two zeros, use the average of the two zeros to find the axis of symmetry.	$\frac{-4 + 0}{2} = \frac{-4}{2} = -2$ Axis of symmetry: $x = -2$	

EXAMPLE 2 Finding the Axis of Symmetry by Using Zeros

Find the axis of symmetry of each parabola.

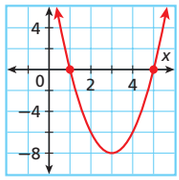
A



$(2, 0)$ *Identify the x-coordinate of the vertex.*

The axis of symmetry is $x = 2$.

B



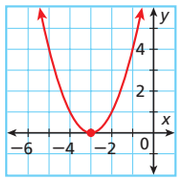
$\frac{1 + 5}{2} = \frac{6}{2} = 3$ *Find the average of the zeros.*

The axis of symmetry is $x = 3$.

CHECK IT OUT!

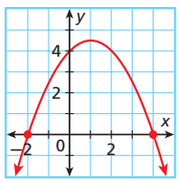
Find the axis of symmetry of each parabola.

2a.



$x = -3$

2b.



$x = 1$

2 Teach

Guided Instruction

Tell students that the zeros of a function can also be called the x-intercepts, or the x-values when $y = 0$. Explain that the x-value of the vertex is the arithmetic mean (average) of the zeros.

Teaching Tip **Geometry** The axis of symmetry is a line of reflection because the figure on the left is a reflection of the figure on the right.

Reaching All Learners

Through Kinesthetic Experience

Have two students hold the ends of a jump rope at the same height from the ground. Use a measuring tape to place the students 4 feet, 6 feet, and 8 feet apart. Point out that the lowest point of the jump rope is always halfway between the students. If there is space in the room, have the two students swing the rope. Show the class that when the rope is overhead, its highest point is also halfway between the two students.

If a function has no zeros or they are difficult to identify from a graph, you can use a formula to find the axis of symmetry. The formula works for all quadratic functions.

Finding the Axis of Symmetry by Using the Formula

FORMULA	EXAMPLE
For a quadratic function $y = ax^2 + bx + c$, the axis of symmetry is the vertical line $x = -\frac{b}{2a}$.	$y = 2x^2 + 4x + 5$ $x = -\frac{b}{2a}$ $= -\frac{4}{2(2)} = -1$ The axis of symmetry is $x = -1$.

EXAMPLE 3 Finding the Axis of Symmetry by Using the Formula

Find the axis of symmetry of the graph of $y = x^2 + 3x + 4$.

Step 1 Find the values of a and b.
$y = 1x^2 + 3x + 4$
$a = 1, b = 3$

Step 2 Use the formula $x = -\frac{b}{2a}$.
$x = -\frac{3}{2(1)} = -\frac{3}{2} = -1.5$

The axis of symmetry is $x = -1.5$.

CHECK IT OUT! **3.** Find the axis of symmetry of the graph of $y = 2x^2 + x + 3$.

$x = -\frac{1}{4}$

Once you have found the axis of symmetry, you can use it to identify the vertex.

Finding the Vertex of a Parabola

Step 1 To find the x-coordinate of the vertex, find the axis of symmetry by using zeros or the formula.

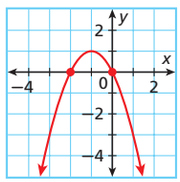
Step 2 To find the corresponding y-coordinate, substitute the x-coordinate of the vertex into the function.

Step 3 Write the vertex as an ordered pair.

EXAMPLE 4 Finding the Vertex of a Parabola

Find the vertex.

A $y = -x^2 - 2x$



Step 1 Find the x-coordinate.
The zeros are -2 and 0.
$x = \frac{-2 + 0}{2} = \frac{-2}{2} = -1$

Step 2 Find the corresponding y-coordinate.
$y = -x^2 - 2x$ *Use the function rule.*
$= -(-1)^2 - 2(-1) = 1$ *Substitute −1 for x.*

Step 3 Write the ordered pair.
$(-1, 1)$

The vertex is $(-1, 1)$.

Caution!
In Example 4A Step 2, use the order of operations to simplify.
$-(-1)^2 = -(1) = -1$

Teaching Tip **Multiple Representations**
Encourage students to use the formula to find the axis of symmetry when the zeros do not appear to be integers.

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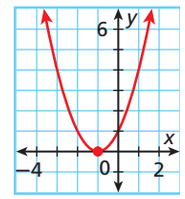
Additional Examples

Example 2

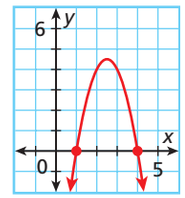
Find the axis of symmetry of each parabola.

A.



$x = -1$

B.



$x = 2.5$

Example 3

Find the axis of symmetry of the graph of $y = -3x^2 + 10x + 9$.

$x = \frac{5}{3}$

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 2

- What does the axis of symmetry tell you about the graph?

EXAMPLE 3

- What values are used in the formula for the axis of symmetry?

Teaching Tip **Inclusion** Remind students that when there appears to be no coefficient for a variable term, the coefficient is 1, and when there is a term missing, the coefficient of the term or the constant is 0.

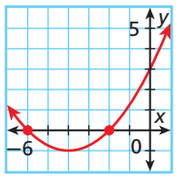
Power Presentations with PowerPoint®

Additional Examples

Example 4

Find the vertex.

A. $y = 0.25x^2 + 2x + 3$ $(-4, -1)$



B. $y = -3x^2 + 6x - 7$ $(1, -4)$

Example 5

The graph of $f(x) = -0.06x^2 + 0.6x + 10.26$ can be used to model the height in meters of an arch support for a bridge, where x represents the horizontal distance in meters from where the arch support enters the water. Can a sailboat that is 14 meters tall pass under the bridge? Explain. Since the highest point of the support is 11.76 m above the water, the sailboat cannot pass under the bridge.

Also available online

Mathematical Practices Questioning Strategies

EXAMPLE 4

- What is the relationship between the vertex and axis of symmetry?

EXAMPLE 5

- What do the zeros represent?
- What part of the graph do you need to find?

Find the vertex.

B $y = 5x^2 - 10x + 3$

Step 1 Find the x-coordinate.

$a = 5$, $b = -10$ *Identify a and b.*

$x = -\frac{b}{2a}$

$= -\frac{-10}{2(5)} = -\frac{-10}{10} = 1$ *Substitute 5 for a and −10 for b.*

The x-coordinate of the vertex is 1.

Step 2 Find the corresponding y-coordinate.

$y = 5x^2 - 10x + 3$ *Use the function rule.*

$= 5(1)^2 - 10(1) + 3$ *Substitute 1 for x.*

$= 5 - 10 + 3$

$= -2$

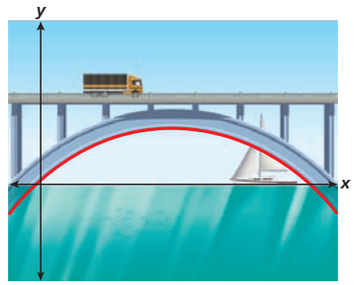
Step 3 Write the ordered pair.

The vertex is $(1, -2)$.

CHECK IT OUT! **4.** Find the vertex of the graph of $y = x^2 - 4x - 10$. $(2, -14)$

EXAMPLE 5 ***Architecture Application***

The height above water level of a curved arch support for a bridge can be modeled by $f(x) = -0.007x^2 + 0.84x + 0.8$, where x is the distance in feet from where the arch support enters the water. Can a sailboat that is 24 feet tall pass under the bridge? Explain.



The vertex represents the highest point of the arch support.

Step 1 Find the x-coordinate.

$a = -0.007$, $b = 0.84$ *Identify a and b.*

$x = -\frac{b}{2a}$

$= -\frac{0.84}{2(-0.007)} = 60$ *Substitute −0.007 for a and 0.84 for b.*

Step 2 Find the corresponding y-coordinate.

$f(x) = -0.007x^2 + 0.84x + 0.8$ *Use the function rule.*

$= -0.007(60)^2 + 0.84(60) + 0.8$ *Substitute 60 for x.*

$= 26$

Since the height of the arch support is 26 feet, the sailboat can pass under the bridge.

CHECK IT OUT! **5.** The height of a small rise in a roller coaster track is modeled by $f(x) = -0.07x^2 + 0.42x + 6.37$, where x is the horizontal distance in feet from a support pole at ground level. Find the greatest height of the rise. 7 ft

3 Close

Summarize

Review how to find the zeros of a function, the vertex, and the axis of symmetry from the graph. Remind students that there may not always be two zeros. Review how to find the axis of symmetry using the formula and how to determine the y-value of the vertex by substituting the value for x in the quadratic function.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

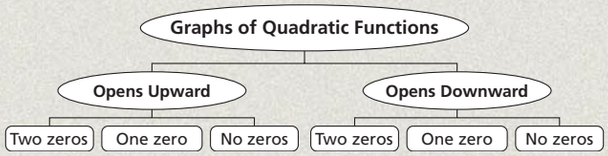
***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. How do you find the zeros of a function from its graph?
2. Describe how to find the axis of symmetry of a quadratic function if its graph does not cross the x-axis
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, sketch a graph that fits the given description.

Know it! Note



8-2 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

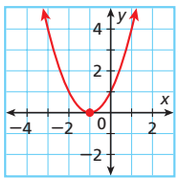
GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

1. Why is the *zero of a function* the same as an x-intercept of a function?
2. Where is the *axis of symmetry* of a parabola located?

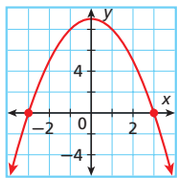
SEE EXAMPLE 1 **Find the zeros of each quadratic function from its graph. Check your answer.**

3. $y = x^2 + 2x + 1$



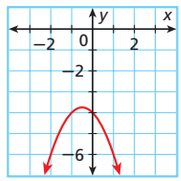
-1

4. $y = 9 - x^2$



$-3, 3$

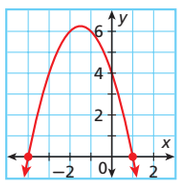
5. $y = -x^2 - x - 4$



no zeros

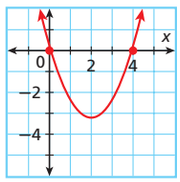
SEE EXAMPLE 2 **Find the axis of symmetry of each parabola.**

6.



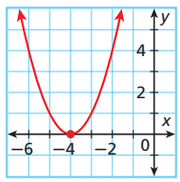
$x = -\frac{3}{2}$

7.



$x = 2$

8.



$x = -4$

SEE EXAMPLE 3 **For each quadratic function, find the axis of symmetry of its graph.**

9. $y = x^2 + 4x - 7$ $x = -2$
10. $y = 3x^2 - 18x + 1$ $x = 3$
11. $y = 2x^2 + 3x - 4$ $x = -\frac{3}{4}$
12. $y = -3x^2 + x + 5$ $x = \frac{1}{6}$

Answers

1. An x-intercept is a value of x where $f(x) = 0$.
2. The axis of symmetry is the vertical line passing through the vertex that divides the parabola into 2 symmetric halves.

Answers to *Think and Discuss*

Possible answers:

1. Find the point(s) where the graph intersects the x-axis. The values of the x-coordinates are the zeros.
2. Using the quadratic function, $y = ax^2 + bx + c$, find the values of a, b, and c. Then use the formula $x = -\frac{b}{2a}$ to find the equation of the axis of symmetry.
3. See Additional Answers.

8-2 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–3
Basic 19–28, 37–39
Average 19–28, 37–39, 44
Advanced 19–28, 37–39, 44

If you finished Examples 1–5
Basic 19–34, 36–43
Average 19–44
Advanced 19–45

Homework Quick Check
Quickly check key concepts.
Exercises: 20, 22, 26, 30, 34

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 18, 34, 36

Construct viable arguments and critique the reasoning of others. Exercises 35, 40

Use appropriate tools strategically. Exercises 37–39, 45

Learn It Online
State Resources Online

MULTI-STEP TEST PREP **Exercise 36** involves finding the vertex of a parabola and telling what it represents. This exercise prepares students for the Multi-Step Test Prep.

SEE EXAMPLE 4

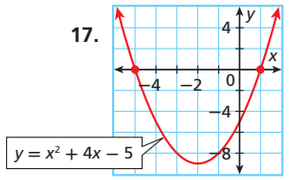
Find the vertex.

13. $y = -5x^2 + 10x + 3$ **(1, 8)**

14. $y = x^2 + 4x - 7$ **(−2, −11)**

15. $y = \frac{1}{2}x^2 + 2x$ **(−2, −2)**

16. $y = -x^2 + 6x + 1$ **(3, 10)**

17. 

(−2, −9)

SEE EXAMPLE 5

18. **Archery** The height in feet above the ground of an arrow after it is shot can be modeled by $y = -16t^2 + 63t + 4$. Can the arrow pass over a tree that is 68 feet tall? Explain. **No; the highest the arrow will go is about 66 feet.**

PRACTICE AND PROBLEM SOLVING

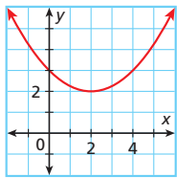
Independent Practice

For Exercises	See Example
19–21	1
22–24	2
25–28	3
29–33	4
34	5

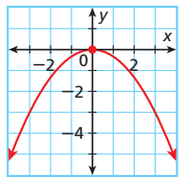
Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

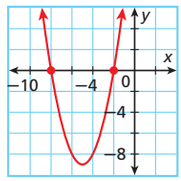
Find the zeros of each quadratic function from its graph. Check your answer.

19. $y = \frac{1}{4}x^2 - x + 3$ 

no zeros

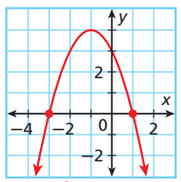
20. $y = -\frac{1}{3}x^2$ 

0

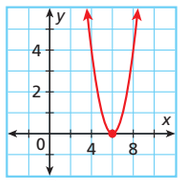
21. $y = x^2 + 10x + 16$ 

−8, −2

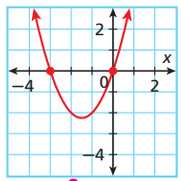
Find the axis of symmetry of each parabola.

22. 

$x = -1$

23. 

$x = 6$

24. 

$x = -\frac{3}{2}$

For each quadratic function, find the axis of symmetry of its graph.

25. $y = x^2 + x + 2$ **$x = -\frac{1}{2}$**

26. $y = 3x^2 - 2x - 6$ **$x = \frac{1}{3}$**

27. $y = \frac{1}{2}x^2 - 5x + 4$ **$x = 5$**

28. $y = -2x^2 + \frac{1}{3}x - \frac{3}{4}$ **$x = \frac{1}{12}$**

LINK Engineering

This arched bridge spans a river near the city of Yokote in northwestern Japan.

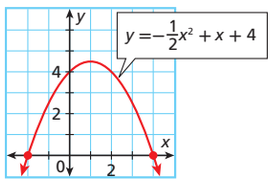
Find the vertex.

29. $y = x^2 + 7x$ **(−3.5, −12.25)**

30. $y = -x^2 + 8x + 16$ **(4, 32)**

31. $y = -2x^2 - 8x - 3$ **(−2, 5)**

32. $y = -x^2 + \frac{1}{2}x + 2$ **$\left(\frac{1}{4}, 2\frac{1}{16}\right)$**

33. 

(1, 4.5)

34. **Yes; the highest point of the arch support is about 8 feet above the creek.**
Engineering The height in feet of the curved arch support for a bridge over a creek can be modeled by $f(x) = -0.628x^2 + 4.5x$, where x is the horizontal distance in feet from where the arch support enters the water. If there is a flood that raises the level of the creek by 5.5 feet, will the top of the arch support be above the water? Explain.

35. **Critical Thinking** What conclusion can be drawn about the axis of symmetry of any quadratic function for which $b = 0$? **The axis of symmetry is the *y*-axis or $x = 0$.**

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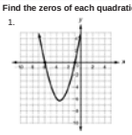
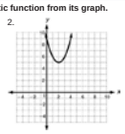
PRACTICE A

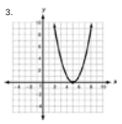
PRACTICE C

PRACTICE B

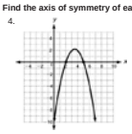
Find the zeros of each quadratic function from its graph.

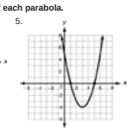
1. 
−6 and 1

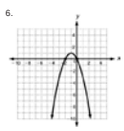
2. 
no zeros

3. 
5

Find the axis of symmetry of each parabola.

4. 
$x = \frac{7}{2}$

5. 
$x = 3$

6. 
$x = -1$

For each quadratic function, find the axis of symmetry of its graph.

7. $y = 3x^2 - 6x + 4$ **$x = 1$**

8. $y = -x^2 + 4x$ **$x = 2$**

9. $y = 4x^2 + \frac{1}{2}x + 3$ **$x = -\frac{1}{16}$**

Find the vertex of each parabola.

10. $y = 3x^2 - 6x - 2$ **(1, −5)**

11. $y = 3x^2 + 12x - 10$ **(−2, −22)**

12. $y = x^2 + 2x - 35$ **(−1, −36)**

READING STRATEGIES

When given the graph of a quadratic function, it is easy to identify its axis of symmetry and vertex. If you are given only the equation, follow the procedure shown below.

$y = \frac{1}{2}x^2 + 4x - 2$

Step 1: Identify a, b, and c.

$a = \frac{1}{2}, b = 4, c = -2$

Step 2: Substitute values into $x = -\frac{b}{2a}$.

$x = -\frac{4}{2\left(\frac{1}{2}\right)} = -\frac{4}{1} = -4$

So, the *axis of symmetry* is the line $x = -4$.

Step 3: Substitute the x-value of the axis of symmetry into the function.

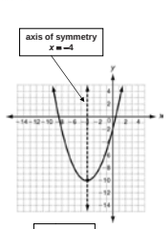
$y = \frac{1}{2}x^2 + 4x - 2$

$y = \frac{1}{2}(-4)^2 + 4(-4) - 2$

$= 8 - 16 - 2$

$= -10$

So, the *vertex* is $(-4, -10)$.



Find the axis of symmetry and vertex for each function.

1. $y = x^2 + 4x + 4$ **$x = -2$, (−2, 0)**

2. $y = 2x^2 + 4x + 1$ **$x = -1$, (−1, −1)**

3. $y = -x^2 + 6x + 5$ **$x = 3$, (3, 14)**

4. $y = -\frac{1}{2}x^2 - 6x + 2$ **$x = -6$, (−6, 20)**

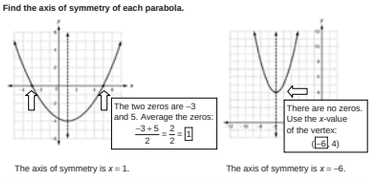
5. $y = 3x^2 - 6x + 1$ **$x = 1$, (1, −2)**

6. $y = x^2 + 10x - 7$ **$x = -5$, (−5, −32)**

RETEACH

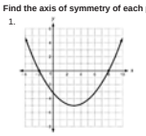
You find the **axis of symmetry** of a parabola by averaging the two zeros. If there is only one zero or no zeros, use the x-value of the vertex.

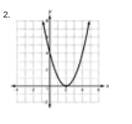
Find the axis of symmetry of each parabola.

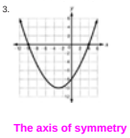


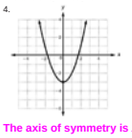
The axis of symmetry is $x = 1$. The axis of symmetry is $x = -6$.

Find the axis of symmetry of each parabola.

1. 
The axis of symmetry is **$x = 3$**.

2. 
The axis of symmetry is **$x = 2$**.

3. 
The axis of symmetry is **$x = -3$**.

4. 
The axis of symmetry is **$x = 0$**.

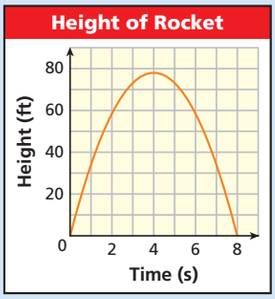
MULTI-STEP TEST PREP

36. **a.** Use the graph of the height of a water bottle rocket to estimate the coordinates of the parabola's vertex.

b. What does the vertex represent?

c. Find the zeros of the function. What do they represent?

d. Find the axis of symmetry. How is it related to the vertex and the zeros?



Graphing Calculator Tell how many zeros each quadratic function has.

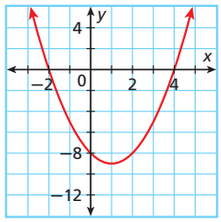
37. $y = 8x^2 - 4x + 2$ **0** **38.** $0 = y + 16x^2$ **1** **39.** $\frac{1}{4}x^2 - 7x - 12 = y - 4$ **2**

40. Write About It If you are given the axis of symmetry of a quadratic function and know that the function has two zeros, how would you describe the location of the two zeros?

The two zeros are on either side of the axis of symmetry and are equidistant from it.

TEST PREP

41. Which function has the zeros shown in the graph?



Ⓐ $y = x^2 + 2x + 8$ Ⓒ $y = x^2 + 2x - 8$

Ⓑ $y = x^2 - 2x - 8$ Ⓓ $y = 2x^2 - 2x + 8$

42. Which of the following functions has a graph with an axis of symmetry of $x = -\frac{1}{2}$?

Ⓕ $y = 2x^2 - 2x + 5$ Ⓗ $2x^2 + y = 2x + 5$

Ⓖ $2x + 5 = 2x^2 - y$ Ⓙ $2x - y = 5 - 2x^2$

43. Gridded Response For the graph of $f(x) = -3 + 20x - 5x^2$, what is the x-coordinate of its vertex? **2**

CHALLENGE AND EXTEND

44. Describe the domain and range of a quadratic function that has exactly one zero and whose graph opens downward.

44. The domain is all real numbers, and the range is $y \leq 0$.

45. Graphing Calculator The height in feet of a parabolic bridge support is modeled by $f(x) = -0.01x^2 + 20$, where $y = -5$ represents ground level and the x-axis represents the middle of the bridge. Find the height and the width of the bridge support. **25 ft; 100 ft**

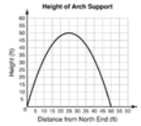
Sam Dudgeon/HMH

PROBLEM SOLVING

Write the correct answer.

1. A superhero is trying to leap over a tall building. The function $f(x) = -16x^2 + 200x$ gives the superhero's height in feet as a function of time. The building is 612 feet high. Will the superhero make it over the building? Explain.
Yes; the vertex is (6.25, 625) and 625 > 612

2. The graph shows the height of an arch support for a pedestrian bridge.



Find the zeros (if any) and axis of symmetry of this parabola.
0 and 50; $x = 25$

3. The distance between the cables suspending a bridge and the water below is given by the function $y = 0.02x^2 - 2x + 80$. Find the vertex of the graph.
(50, 30)

After a heavy snowfall, Joe and Karin made an igloo. The dome of the igloo is in the shape of a parabola and the height of the igloo in inches is given by $f(x) = -0.03x^2 + 2.4x$. Select the best answer.

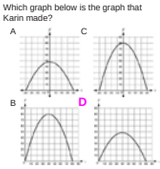
4. Joe wants to place a support in the middle of the igloo, along the axis of symmetry. How far from the edge of the igloo should he place the support?
A 24 in. C 48 in.
B 40 in. D 80 in.

5. Neither Joe nor Karin can stand up inside the igloo. How tall is the center of the igloo? (*Hint:* the top of the igloo is the vertex of the parabola.)
F 24 in. **H** 48 in.
G 40 in. J 80 in.

6. Karin graphs the parabola and looks at the zeros to see how wide the igloo is. What are the zeros of this parabola?
A −80 and 80 **C** 0 and 80
B −40 and 40 D 40 and 80

7. What is the vertex of the parabola that Karin graphed?
F (20, 36) H (48, 40)
G (40, 48) J (80, 0)

8. Which graph below is the graph that Karin made?



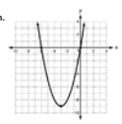
CHALLENGE

You may recall from Chapter 5 that for any *two* points, there is exactly one *line* that contains both. You used this property to find the equation of a line that passes through two given points.

A similar property exists for parabolas — for any *three* points, there is exactly one *parabola* that contains all three. You'll use this property to find the quadratic function that passes through three given points.

Follow these steps to find the quadratic function for this graph.

1. Name the coordinates of the two x-intercepts and the vertex.
(−6, 0) (−3, −9) (0, 0)



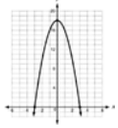
2. Substitute each pair of coordinates into the general quadratic function $y = ax^2 + bx + c$, and simplify. You will get three *linear* equations with the variables a, b, and c.
$0 = 36a - 6b + c$
$-9 = 9a - 3b + c$
$0 = c$

3. Treat the three linear equations in question 2 as a system. Use substitution and elimination to find the values of a, b, and c.
$a = 1$, $b = 6$, $c = 0$

4. Substitute the values of a, b, and c into the general function $y = ax^2 + bx + c$. What is the specific quadratic function for the graph above? $y = x^2 + 6x$

Use a similar process to find these quadratic functions.

5. The quadratic function for the graph at right.
$0 = 9a - 3b + c$
$18 = c$; $y = -2x^2 + 18$
$0 = 9a + 3b + c$



6. The quadratic function that passes through the points (−5, 0), (2, 0), and (−1.5, −12.25).
$0 = 25a - 5b + c$
$0 = 4a + 2b + c$; $y = x^2 + 3x - 10$
$-12.25 = 2.25a - 1.5b + c$

TEST PREP DOCTOR In **Exercise 41,** remind students that they can determine the y-intercept by substituting 0 for x. Students can immediately eliminate **A** and **D** because the graphs of those functions cross the y-axis at 8, not −8.

Answers

36a. about (4, 78)

b. the time and height at the moment when the rocket was at its highest point

c. 0 and 8; the starting and ending times in seconds

d. $x = 4$; the x-value of the axis of symmetry is the average of the two zeros; it is also the x-coordinate of the vertex.

Journal

Have students explain how the vertex and the axis of symmetry are related graphically and algebraically.

ALTERNATIVE ASSESSMENT

Have students create two quadratic functions: one with one zero and the other with two zeros. Have them state the zeros, the axis of symmetry, and the vertex for each.

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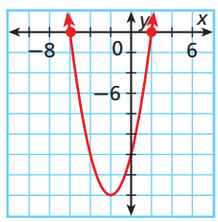
8-2 Lesson Quiz

1. Find the zeros and the axis of symmetry of the parabola.



zeros: −6, 2; $x = -2$

2. Find the axis of symmetry and the vertex of the graph of $y = 3x^2 + 12x + 8$. $x = -2$; $(-2, -4)$

3. The graph of $f(x) = -0.01x^2 + x$ can be used to model the height in feet of a curved arch support for a bridge, where x represents the horizontal distance in feet from where the arch support enters the water. Find the height of the highest point of the support. 25 ft

Also available online

8-3 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Graph a quadratic function in the form $y = ax^2 + bx + c$.

Algebra Lab
In *Algebra Lab Activities*

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Find the axis of symmetry.

1. $y = 4x^2 - 7$ $x = 0$
2. $y = x^2 - 3x + 1$ $x = \frac{3}{2}$
3. $y = -2x^2 + 4x + 3$ $x = 1$
4. $y = -2x^2 + 3x - 1$ $x = \frac{3}{4}$

Find the vertex.

5. $y = x^2 + 4x + 5$ $(-2, 1)$
6. $y = 3x^2 + 2$ $(0, 2)$
7. $y = 2x^2 + 2x - 8$ $\left(-\frac{1}{2}, -\frac{17}{2}\right)$

Also available online

Math Humor

Q: What do you call a tough old quadratic equation?

A: A battle-ax^2.

COMMON CORE

CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
Also **CC.9-12.F.IF.4*, CC.9-12.F.IF.8**

Learn It Online
State Resources Online

COMMON CORE

8-3

Graphing Quadratic Functions

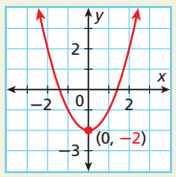
CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, …. *
a. Graph … quadratic functions and show intercepts, maxima, and minima. *Also* **CC.9-12.F.IF.4*, CC.9-12.F.IF.8**

Objective
Graph a quadratic function in the form $y = ax^2 + bx + c$.

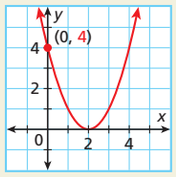
Why use this?
Graphs of quadratic functions can help you determine how high an object is tossed or kicked. (See Exercise 14.)

Recall that a y-intercept is the y-coordinate of the point where a graph intersects the y-axis. The x-coordinate of this point is always 0. For a quadratic function written in the form $y = ax^2 + bx + c$, when $x = 0$, $y = c$. So the y-intercept of a quadratic function is c.

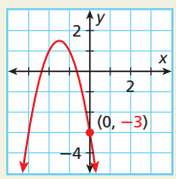
$y = x^2 - 2$
$y = x^2 + (-2)$



$y = x^2 - 4x + 4$



$y = -2x^2 - 6x - 3$
$y = -2x^2 - 6x + (-3)$



Previously, you found the axis of symmetry and vertex of a parabola. You can use these characteristics, the y-intercept, and symmetry to graph a quadratic function.

EXAMPLE 1 Graphing a Quadratic Function

Graph $y = x^2 - 4x - 5$.

Step 1 Find the axis of symmetry.

$x = -\frac{-4}{2(1)}$ *Use $x = -\frac{b}{2a}$. Substitute 1 for a and −4 for b.*

$= 2$ *Simplify.*

The axis of symmetry is $x = 2$.

Step 2 Find the vertex.

$y = x^2 - 4x - 5$

$= 2^2 - 4(2) - 5$ *The x-coordinate of the vertex is 2. Substitute 2 for x.*

$= 4 - 8 - 5$ *Simplify.*

$= -9$ *The y-coordinate is −9.*

The vertex is $(2, -9)$.

Step 3 Find the y-intercept.

$y = x^2 - 4x - 5$

$y = x^2 - 4x + (-5)$ *Identify c.*

The y-intercept is -5; the graph passes through $(0, -5)$.

Michael S. Yamashita/CORBIS

1 Introduce

EXPLORATION

Graphing Quadratic Functions

Recall that a y-intercept is the y-coordinate of a point where a graph intersects the y-axis. Use a graphing calculator to explore the y-intercepts of some quadratic functions.

1. Use your calculator to graph each of the quadratic functions in the table. For each function, use the graph to identify the y-intercept.

Function	y-intercept
$y = x^2 - 2x + 3$	3
$y = x^2 + x - 6$	−6
$y = -2x^2 + 3x + 4$	4
$y = -\frac{1}{2}x^2 - x - 2$	−2
$y = 3x^2 - 5x$	0

2. Look for a pattern in your table. In general, what can you say about the y-intercept of a quadratic function written in the form $y = ax^2 + bx + c$? The y-intercept is c.

3. Without graphing the function, determine the y-intercept of $y = 4x^2 - 7x - 8$. −8

THINK AND DISCUSS

4. **Explain** how knowing the y-intercept of a quadratic function can help you sketch its graph. The y-intercept tells you where the graph crosses the y-axis.

Motivate

Discuss with students how the more they know about the properties of a geometric figure, the easier it is for them to sketch an accurate drawing of it. For example, if you know the length of a rectangle, you have only part of the information you need to draw it accurately. If you also know the width of the rectangle, then you can make an accurate drawing. Tell students that in this lesson they will learn to use the properties of parabolas to graph them.

Explorations and answers are provided in your online resources.

Step 4 Find two more points on the same side of the axis of symmetry as the point containing the *y*-intercept.

Since the axis of symmetry is $x = 2$, choose *x*-values less than 2.

Let $x = 1$.

$y = 1^2 - 4(1) - 5$ *Substitute x-coordinates.*

$= 1 - 4 - 5$ *Simplify.*

$= -8$

Let $x = -1$.

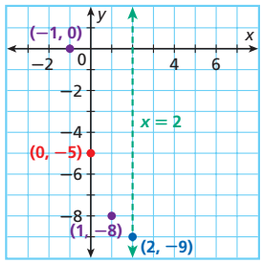
$y = (-1)^2 - 4(-1) - 5$

$= 1 + 4 - 5$

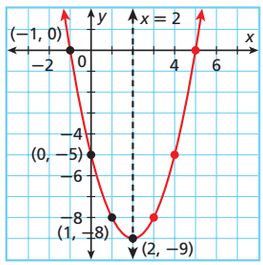
$= 0$

Two other points are $(1, -8)$ and $(-1, 0)$.

Step 5 Graph the **axis of symmetry**, the **vertex**, the point containing the ***y*-intercept**, and **two other points**.



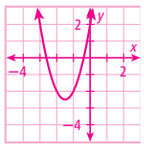
Step 6 **Reflect** the points across the axis of symmetry. Connect the points with a smooth curve.



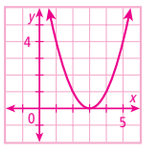
Helpful Hint

Because a parabola is symmetrical, each point is the same number of units away from the axis of symmetry as its reflected point.

1a.



1b.



CHECK IT OUT! **Graph each quadratic function.**

1a. $y = 2x^2 + 6x + 2$

1b. $y + 6x = x^2 + 9$

EXAMPLE 2 *Problem-Solving Application*

The height in feet of a football that is kicked can be modeled by the function $f(x) = -16x^2 + 64x$, where *x* is the time in seconds after it is kicked. Find the football's maximum height and the time it takes the football to reach this height. Then find how long the football is in the air.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

Remember!

The vertex is the highest or lowest point on a parabola. Therefore, in the example, it gives the maximum height of the football.

1 Understand the Problem

The **answer** includes three parts: the maximum height, the time to reach the maximum height, and the time to reach the ground.

List the important information:

- The function $f(x) = -16x^2 + 64x$ models the approximate height of the football after *x* seconds.

2 Make a Plan

Find the vertex of the graph because the maximum height of the football and the time it takes to reach it are the coordinates of the vertex. The football will hit the ground when its height is 0, so find the zeros of the function. You can do this by graphing.

COMMON ERROR ALERT

Some students get confused by multiple negative signs when using the formula to find the axis of symmetry. Encourage them to write the expression as $-\left(\frac{b}{2a}\right)$.

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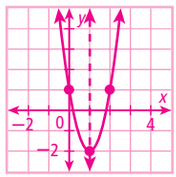
Additional Examples

Example 1

Graph $y = 3x^2 - 6x + 1$.



Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 1

- How do you know whether the parabola opens upward or downward?
- Can you have points both above and below the vertex?
- What does the axis of symmetry tell you about the graph?

2 Teach

Guided Instruction

Review the characteristics of a parabola with students. Show students how to use these characteristics to graph the parabola. Then show how to interpret the graph of a quadratic function when it models a real-world relationship such as height over time.

Reaching All Learners *Through Modeling*

Have students in small groups create a scenario that could be represented by a quadratic function (see examples). Have them create a graph of their function using realistic labels and scales on the axes (they do not need to create a function rule) and present it to the class. They should tell the class what scenario is modeled by the graph, what the vertex and zeros represent, and what a reasonable domain and range are.

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Additional Examples

Example 2

The height in feet of a soccer ball can be modeled by $f(x) = -16x^2 + 32x$, where x is the time in seconds after it is kicked. Find the soccer ball's maximum height and the time it takes the soccer ball to reach this height. Then find how long the soccer ball is in the air. 16 ft; 1 s; 2 s

Also available online

Questioning Strategies

EXAMPLE 2

- What do the zeros represent in the problem?
- What is a reasonable domain and range in this situation?

Reading Math Emphasize that the quadratic model in **Example 2** models height over time. Point out that a height-over-time model does not represent the shape of the projectile's path and that the units on the axes are not both measures of length.

Multiple Representations Students can check the reasonableness of their graph by choosing a reflected point and substituting the ordered pair into the function. If it satisfies the function, the point was reflected correctly and the graph is reasonable.

3 Solve

Step 1 Find the axis of symmetry.

$x = -\frac{64}{2(-16)}$ *Use $x = -\frac{b}{2a}$. Substitute −16 for a and 64 for b.*

$= -\frac{64}{-32} = 2$ *Simplify.*

The axis of symmetry is $x = 2$.

Step 2 Find the vertex.

$y = -16x^2 + 64x$

$= -16(2)^2 + 64(2)$ *The x-coordinate of the vertex is 2. Substitute 2 for x.*

$= -16(4) + 128$ *Simplify.*

$= -64 + 128$

$= 64$ *The y-coordinate is 64.*

The vertex is $(2, 64)$.

Step 3 Find the y-intercept.

$y = -16x^2 + 64x + 0$ *Identify c.*

The y-intercept is 0; the graph passes through $(0, 0)$.

Step 4 Find another point on the same side of the axis of symmetry as the point containing the y-intercept.

Since the axis of symmetry is $x = 2$, choose an x-value that is less than 2.

Let $x = 1$.

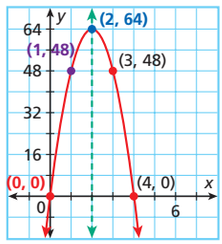
$y = -16(1)^2 + 64(1)$ *Substitute 1 for x.*

$= -16 + 64$ *Simplify.*

$= 48$

Another point is $(1, 48)$.

Step 5 Graph the axis of symmetry, the vertex, the point containing the y-intercept, and the other point. Then reflect the points across the axis of symmetry. Connect the points with a smooth curve.



The vertex is $(2, 64)$. So at 2 seconds, the football has reached its maximum height of 64 feet. The graph shows the zeros of the function are 0 and 4. At 0 seconds the football has not yet been kicked, and at 4 seconds it reaches the ground. The football is in the air for 4 seconds.

4 Look Back

Check by substituting $(2, 64)$ and $(4, 0)$ into the function.

$64 = -16(2)^2 + 64(2)$ $\quad$ $0 = -16(4)^2 + 64(4)$

$64 = -64 + 128$ $\quad$ $0 = -256 + 256$

$64 = 64$ ✓ $\quad$ $0 = 0$ ✓

CHECK IT OUT! **2.** As Molly dives into her pool, her height in feet above the water can be modeled by the function $f(x) = -16x^2 + 16x + 12$ where x is the time in seconds after she begins diving. Find the maximum height of her dive and the time it takes Molly to reach this height. Then find how long it takes her to reach the pool. **maximum height: 16 ft at 0.5 s; time it takes to reach the pool: 1.5 s**

3 Close

Summarize

Have students identify the vertex of each function. Then have them tell whether the parabola opens upward or downward.

$y = 4x^2 - 4$ $(0, -4)$; upward

$y = 2x^2 + 2x + 1$ $\left(-\frac{1}{2}, \frac{1}{2}\right)$; upward

$y = -x^2 + 6x - 7$ $(3, 2)$; downward

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

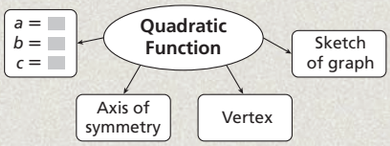
THINK AND DISCUSS

MATHEMATICAL PRACTICES

1. Explain how to find the y-intercept of a quadratic function that is written in the form $ax^2 - y = bx + c$.
2. Explain how to graph a quadratic function.
3. What do the vertex and zeros of the function represent in the situation in the Check It Out for Example 2?

Know it! Note

4. **GET ORGANIZED** Copy and complete the graphic organizer using your own quadratic function.



8-3 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

SEE EXAMPLE 1

Graph each quadratic function.

1. $y = x^2 - 2x - 3$
2. $-y - 3x^2 = -3$
3. $y = 2x^2 + 2x - 4$
4. $y = x^2 + 4x - 8$
5. $y + x^2 + 5x + 2 = 0$
6. $y = 4x^2 + 2$

SEE EXAMPLE 2

7. **Multi-Step** The height in feet of a golf ball that is hit from the ground can be modeled by the function $f(x) = -16x^2 + 96x$, where x is the time in seconds after the ball is hit. Find the ball's maximum height and the time it takes the ball to reach this height. Then find how long the ball is in the air.

maximum height: 144 ft at 3 s; time in the air: 6 s

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
8–13	1
14	2

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

Graph each quadratic function.

8. $y = -4x^2 + 12x - 5$
9. $y = 3x^2 + 12x + 9$
10. $y - 7x^2 - 14x = 3$
11. $y = -x^2 + 2x$
12. $y - 1 = 4x^2 + 8x$
13. $y = -2x^2 - 3x + 4$
14. **Multi-Step** A juggler tosses a ring into the air. The height of the ring in feet above the juggler's hands can be modeled by the function $f(x) = -16x^2 + 16x$, where x is the time in seconds after the ring is tossed. Find the ring's maximum height above the juggler's hands and the time it takes the ring to reach this height. Then find how long the ring is in the air.

4 ft at 0.5 s; 1 s

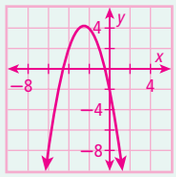
15. $x = 4$; $(4, -16)$

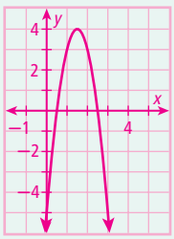
For each quadratic function, find the axis of symmetry and the vertex of its graph.

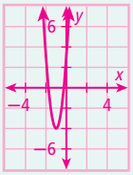
15. $y = x^2 - 8x$
16. $y = -x^2 + 6x - 4$
17. $y = 4 - 3x^2$ $x = 0$; $(0, 4)$
18. $y = -2x^2 - 4$ $x = 0$; $(0, -4)$
19. $y = -x^2 - x - 4$
20. $y = x^2 + 8x + 16$ $x = -4$; $(-4, 0)$

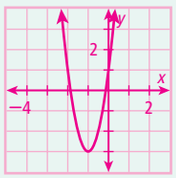
16. $x = 3$; $(3, 5)$

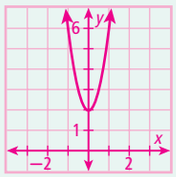
19. $x = -\frac{1}{2}$; $\left(-\frac{1}{2}, -\frac{15}{4}\right)$

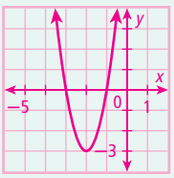
5. 

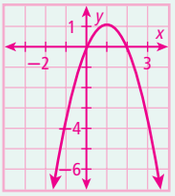
8. 

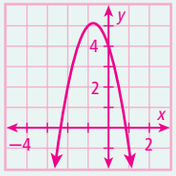
10. 

12. 

6. 

9. 

11. 

13. 

Answers to *Think and Discuss*

Possible answers:

1. Solve the equation for y. The resulting value of the constant term is the y-intercept.
2. Find the axis of symmetry, the vertex, the point containing the y-intercept, and several other points on the graph. Draw a curved line that passes through the points and is symmetrical about the axis of symmetry.
3. The vertex shows the maximum height of Molly's dive and the time at which she reached this height. The zero shows the time when she reaches the pool.
4. See Additional Answers.

8-3 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

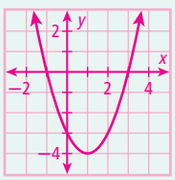
If you finished Examples **1–2**

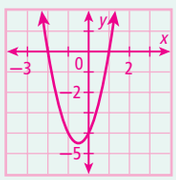
Basic 8–28, 35–40
Average 8–41
Advanced 8–42

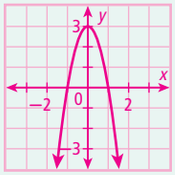
Homework Quick Check

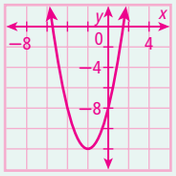
Quickly check key concepts.
Exercises: 8, 10, 14, 16, 24, 27

Answers

1. 

3. 

2. 

4. 

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 7, 14, 27, 30–32, 35

Construct viable arguments and critique the reasoning of others. Exercises 28–29, 33–34

MULTI-STEP TEST PREP **Exercise 35** involves writing and using a quadratic model for projectile motion. This exercise prepares students for the Multi-Step Test Prep.

Teaching Tip **Inclusion** Students may be overwhelmed by the number of variables used in **Exercise 35.** Help them understand what each variable represents: h is the dependent variable, and t is the independent variable. Then help students relate v_i to the coefficient b and h_i to the constant c. Show them that $-\frac{1}{2}a$ is the leading coefficient.

Answers

21–27a. See Additional Answers.

33. $(-1, 6)$; the axis of symmetry is a vertical line through the vertex. So its equation is $x = 0$. Reflect the point $(1, 6)$ across the axis of symmetry.

34. The x-coordinate of the vertex gives the axis of symmetry. If the range includes values greater than the y-coordinate of the vertex, then the parabola opens upward. If the range includes values less than the y-coordinate of the vertex, then the parabola opens downward.

35c–d. See Additional Answers.

LINK Travel

Building began on the Tower of Pisa, located in Pisa, Italy, in 1173. The tower started leaning after the third story was added. At the fifth story, attempts were made to correct the leaning. The tower was finally complete in 1350.

Graph each quadratic function. On your graph, label the coordinates of the vertex. Draw and label the axis of symmetry.

21. $y = -x^2$ **22.** $y = -x^2 + 4x$ **23.** $y = x^2 - 6x + 4$

24. $y = x^2 - x$ **25.** $y = 3x^2 - 4$ **26.** $y = -2x^2 - 16x - 25$

27. Travel While on a vacation in Italy, Rudy visited the Leaning Tower of Pisa. When he leaned over the railing to look down from the tower, his sunglasses fell off. The height in meters of the sunglasses as they fell can be approximated by the function $y = -5x^2 + 50$, where x is the time in seconds.

a. Graph the function. (*Hint:* Use a graphing calculator.) **27b. D: $0 \le x \le 3.16$; R: $0 \le y \le 50$**

b. What is a reasonable domain and range?

c. How long did it take for the glasses to reach the ground? **about 3.16 s**

28. ERROR ANALYSIS Two students found the equation of the axis of symmetry for the graph of $f(x) = -x^2 - 2x + 1$. Who is incorrect? Explain the error.

A

$x = -\frac{b}{2a}$

$x = -\frac{-1}{2(-2)} = -\frac{1}{4}$

Axis of symmetry is $x = -\frac{1}{4}$.

B

$x = -\frac{b}{2a}$

$x = -\frac{-2}{2(-1)} = -\frac{-2}{-2} = -1$

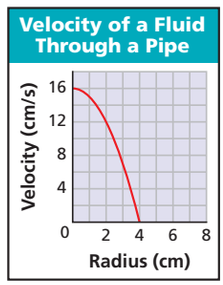
Axis of symmetry is $x = -1$.

28. Student A is incorrect; the student incorrectly identified the values *a* and *b*.

29. Critical Thinking The point $(5, 4)$ lies on the graph of a quadratic function whose axis of symmetry is $x = 2$. Find another point on the graph. Explain how you found the point.

29. $(-1, 4)$; reflect the given point across the axis of symmetry.

Engineering **Use the graph for Exercises 30–32. The velocity v in centimeters per second of a fluid flowing in a pipe varies according to the radius r of the pipe.**



30. Find the radius of the pipe when the velocity is 7 cm/s. **3 cm**

31. Find the velocity of the fluid when the radius is 2 cm. **12 cm/s**

32. What is a reasonable domain for this function? Explain.

32. $0 \le x \le 4$; when x is greater than 4, the velocity would be negative, which does not make sense.

33. Critical Thinking The graph of a quadratic function has the vertex $(0, 5)$. One point on the graph is $(1, 6)$. Find another point on the graph. Explain how you found the point.

34. Write About It Explain how the vertex and the range can help you graph a quadratic function.

MULTI-STEP TEST PREP

35. A water bottle rocket is shot upward with an initial velocity of $v_i = 45$ ft/s from the roof of a school, which is at h_i, 50 ft above the ground. The equation $h = -\frac{1}{2}at^2 + v_i t + h_i$ models the rocket's height as a function of time. The acceleration due to gravity a is 32 ft/s^2. **$h(t) = -16t^2 + 45t + 50$**

a. Write the equation for height as a function of time for this situation.

b. Find the vertex of this parabola. **approximately $(1.4, 81.6)$**

c. Sketch the graph of this parabola and label the vertex.

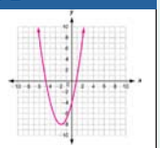
d. What do the coordinates of the vertex represent in terms of time and height?

(t), AP Photo/Fabio Muzzi; (b), Sam Dudgeon/HMH

PRACTICE A
PRACTICE C
PRACTICE B

Graph each quadratic function.

1. $y = x^2 + 4x - 4$



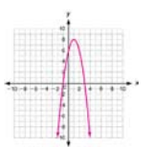
axis of symmetry: $x = -2$

vertex: $(-2, -8)$

y-intercept: -4

two other points: Possible answers: $(1, 1)$ and $(2, 8)$

2. $y + 2x^2 - 4x - 6 = 0$



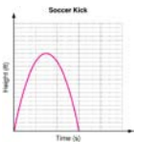
axis of symmetry: $x = 1$

vertex: $(1, 8)$

y-intercept: 6

two other points: Possible answers: $(-1, 0)$ and $(-2, -10)$

3. The height in feet of a soccer ball that is kicked can be modeled by the function $f(x) = -8x^2 + 24x$, where x is the time in seconds after it is kicked. Graph this function. Find the soccer ball's maximum height and the time it takes the ball to reach this height. Then find how long the soccer ball is in the air.



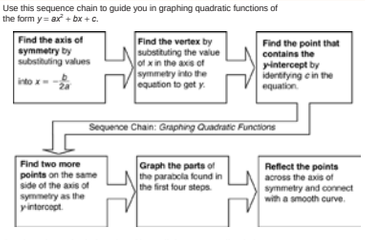
maximum height: 18 feet

time to reach maximum height: 1.5 seconds

time in the air: 3 seconds

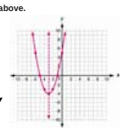
READING STRATEGIES

Use this sequence chain to guide you in graphing quadratic functions of the form $y = ax^2 + bx + c$.



Graph each quadratic function. Show all the steps outlined above.

1. $y = x^2 + 6x + 5$



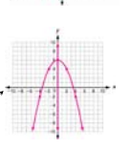
Step 1: $x = -3$

Step 2: $(-3, -4)$

Step 3: $(0, 5)$

Step 4: Possible answers: Step 5 and Step 6 $(-2, -3)$ and $(-1, 0)$

2. $y = -\frac{1}{2}x^2 + 6$



Step 1: $x = 0$

Step 2: $(0, 6)$

Step 3: $(0, 6)$

Step 4: Possible answers: Step 5 and Step 6 $(-2, 4)$ and $(-4, -2)$

RETEACH

You can use the axis of symmetry, vertex, and y-intercept to graph a quadratic function.

Graph $y = x^2 + 6x + 8$.

Step 1: Find the axis of symmetry.

$x = -\frac{6}{2(1)} = -3$ Use $x = -\frac{b}{2a}$

Graph the axis of symmetry, $x = -3$.

Step 2: Find the vertex.

$y = (-3)^2 + 6(-3) + 8$ *Substitute −3 for x.*

$y = 9 - 18 + 8$

$y = -1$ *Simplify.*

Graph the vertex, $(-3, -1)$.

Step 3: Find the y-intercept.

$y = (0)^2 + 6(0) + 8$ *Substitute 0 for x in the original equation.*

$y = 8$ *Simplify.*

Graph $(0, 8)$.

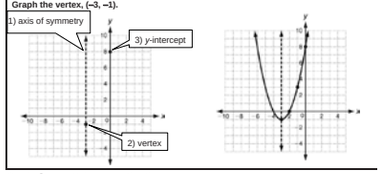
Step 4: Choose two x-values on the same side of the axis of symmetry as the point containing the y-intercept.

Use −2 and −1.

$y = (-2)^2 + 6(-2) + 8 = 0$ **Graph $(-2, 0)$.**

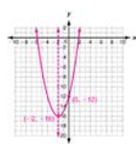
$y = (-1)^2 + 6(-1) + 8 = 3$ **Graph $(-1, 3)$.**

Step 5: Reflect those points and connect them with a smooth curve.



Graph $x^2 + 4x - 12$ by completing the following.

1. Find and graph the axis of symmetry.
   $x = -2$
2. Find and graph the vertex.
   $(-2, -16)$
3. Find and graph the y-intercept.
   $(0, -12)$
4. Find and graph two more points.
   Possible answers: $(-1, -15)$; $(1, -7)$
5. Reflect the points and draw the graph.



36. Copy and complete the table for each function.

Function	Graph Opens	Axis of Symmetry	Vertex	Zeros	Domain and Range
$y = x^2 + 4$	Upward	$x = 0$	$(0, 4)$	None	D: all real numbers R: $y \geq 4$
$y = -x^2 + 4$	Downward	$x = 0$	$(0, 4)$	$-2, 2$	D: all real numbers R: $y \leq 4$
$y + 8 - x^2 = -2x$	Upward	$x = 1$	$(1, -9)$	$-2, 4$	D: all real numbers R: $y \geq -9$

TEST PREP

37. Which is the axis of symmetry for the graph of $f(x) = 6 - 5x + \frac{1}{2}x^2$?

(A) $x = 5$ (B) $x = \frac{1}{20}$ (C) $x = -5$ (D) $x = -\frac{1}{20}$

38. What are the coordinates of the vertex for the graph of $f(x) = x^2 - 5x + 6$?

(F) $\left(-\frac{5}{2}, -\frac{1}{4}\right)$ (G) $\left(-\frac{5}{2}, \frac{1}{4}\right)$ (H) $\left(\frac{5}{2}, \frac{1}{4}\right)$ (J) $\left(\frac{5}{2}, -\frac{1}{4}\right)$

39. Which function's graph has an axis of symmetry of $x = 1$ and a vertex of $(1, 8)$?

(A) $y = -x^2 + x + 8$ (C) $y = 2x^2 - 4x - 8$

(B) $y = x^2 + 8x + 1$ (D) $y = -3x^2 + 6x + 5$

40. Short Response Graph $y = x^2 + 3x + 2$. What are the zeros, the axis of symmetry, and the coordinates of the vertex? Show your work.

CHALLENGE AND EXTEND

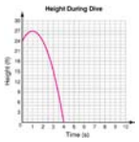
41. The graph of a quadratic function has its vertex at $(1, -4)$ and one zero of the function is 3. Find the other zero. Explain how you found the other zero.

42. The x-intercepts of a quadratic function are 3 and -3. The y-intercept is 6. What are the coordinates of the vertex? Does the function have a maximum or a minimum? Explain.

PROBLEM SOLVING

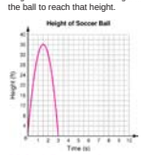
Write the correct answer.

1. An Olympic diver is competing for a medal. His height in feet above the water can be modeled by the function $f(x) = -3x^2 + 6x + 24$, where x is the time in seconds after he begins the dive. Graph the function. Then find how long it takes the diver to reach the water.



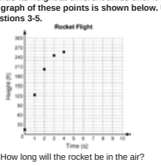
4 seconds

2. Tanisha kicks a soccer ball during a game. The height of the ball, in feet, can be modeled by the function $f(x) = -16x^2 + 48x$, where x is the time in seconds after she kicks the ball. Graph the function. Find the maximum height of the ball and how long it takes the ball to reach that height.



36 ft; 1.5 seconds

A model rocket is launched from a platform into the air. Keona records its height at different times until it reaches its peak at 259 ft. Her graph of these points is shown below. Use this graph to answer questions 3-5.



3. Keona wants to complete her graph by plotting the heights of the rocket as it descended. Which of the following points will she graph?

F (4, 180) H (6, 200)
G (5, 150) J (10, 0)

4. How long will the rocket be in the air?

A 4 seconds C 8 seconds
B 6 seconds D 10 seconds

5. Which of the following equations models the flight of the ball where x is the time in seconds and y is the height in feet?

A $16x^2 + 15x + 125$
B $16x^2 + 125x + 15$
C $-16x^2 + 125x + 15$
D $-16x^2 + 15x + 125$

CHALLENGE

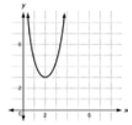
You can find an equation for the axis of symmetry of a parabola by using the coefficients of an equation in the form $y = ax^2 + bx + c$.

Let $y = 2x^2 - 8x + 11$. In Exercises 1–4, refer to the graph shown at right.

1. Complete each table of values below.

x	−1	0	1	2	3	4	5
y	21	11	5	3	5	11	21

x	1.7	1.8	1.9	2.0	2.1	2.2	2.3
y	3.18	3.08	3.02	3.00	3.02	3.08	3.18



2. Write the coordinates of the vertex of the parabola. (2, 3)
3. Using your conclusions from Exercises 1 and 2, write an equation for the axis of symmetry. $x = 2$
4. Find the value of $\frac{-b}{2a}$ in $y = 2x^2 - 8x + 11$. 2

Let $y = -3x^2 - 6x - 5$.

5. Complete the table of values below.

x	−1.3	−1.2	−1.1	−1.0	−0.9	−0.8	−0.7
y	−2.27	−2.12	−2.03	−2.00	−2.03	−2.12	−2.27

6. Write the coordinates of the vertex of the parabola. (−1, −2)
7. Using the table in Exercise 5, write an equation for the axis of symmetry. $x = -1$
8. Find the value of $\frac{-b}{2a}$ in $y = -3x^2 - 6x - 5$. −1

Let $y = ax^2 + bx + c$.

9. a. Write a conjecture that gives an equation for the axis of symmetry of the parabola in terms of the coefficients of $y = ax^2 + bx + c$.

An equation for the axis of symmetry of the graph of $y = ax^2 + bx + c$ is $x = \frac{-b}{2a}$.

b. Explain how to verify the conjecture that you wrote in part **a**.

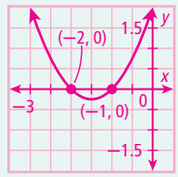
One approach is to use graphs, and a second approach is to use tables as in Exercises 1 and 5.

TEST PREP DOCTOR Suggest that students rewrite the function in **Exercise 37** in standard form as a first step.

In **Exercise 38,** students can eliminate choices **F** and **G** before finding the axis of symmetry by counting the negative signs of $-\frac{b}{2a}$.

Answers

40.



zeros: $-2, -1$
axis of symmetry: $x = -\frac{3}{2}$
vertex: $\left(-\frac{3}{2}, -\frac{1}{4}\right)$

41. -1; the axis of symmetry is $x = 1$. The given zero is 2 units right of the axis of symmetry. The other zero is 2 units left of the axis of symmetry.

Journal

Have students compare and contrast the process for graphing a linear function with the process for graphing a quadratic function.

ALTERNATIVE ASSESSMENT

Have students write and graph a quadratic function in the form $y = ax^2 + bx + c$, where a, b, and c are not zero. Have students give the domain and range of the function.

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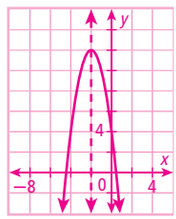
8-3 Lesson Quiz

1. Graph $y = -2x^2 - 8x + 4$.



2. The height in feet of a fireworks shell can be modeled by $h(t) = -16t^2 + 224t$, where t is the time in seconds after it is fired. Find the maximum height of the shell, the time it takes to reach its maximum height, and the length of time the shell is in the air if it does not explode. 784 ft; 7 s; 14 s

Also available online

Organizer

Use with the lesson *Transforming Quadratic Functions*

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use a graphing calculator to explore how the value of a affects the graph of $y = ax^2$.

Materials: graphing calculator

Online Edition
Graphing Calculator, TechKeys

Countdown

Resources

Technology Lab Activities
Lab Recording Sheet

Teach

Discuss

Encourage students to test other values for a, such as 0.1, 0.2, and 0.3.

Close

Key Concept

The absolute value of a affects the width of the graph of $y = ax^2$. The sign of a affects whether the graph opens upward or downward.

Assessment

Journal Have students explain how the graphs of $y = 5x^2$ and $y = \frac{1}{5}x^2$ compare with the graph of $y = x^2$.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Also* **CC.9-12.F.IF.7***

Learn It Online
State Resources Online

8-4 Technology LAB

Use with Transforming Quadratic Functions

The Family of Quadratic Functions

Functions whose graphs share the same basic characteristics form a *family of functions*. All quadratic functions form a family because their graphs are all parabolas. You can use a graphing calculator to explore the family of quadratic functions.

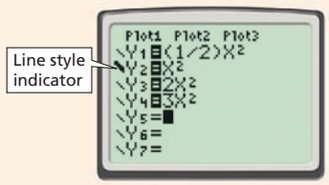
Learn It Online
Lab Resources Online

Activity

Describe how the value of a affects the graph of $y = ax^2$.

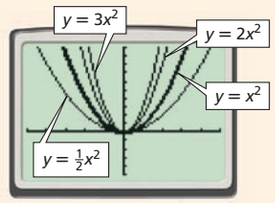
1. Press Y=. Enter Y_1 through Y_4 as shown.

   Notice that Y_2 represents the parent function $y = x^2$. To make it stand out from the other functions, change its line style. When you enter Y_2, move the cursor to the line style indicator by pressing ◀. Then press ENTER to cycle through the choices until a thicker line appears.



2. Press GRAPH.

   Keep in mind the values of a as the functions are graphed. The graphing calculator will graph the functions in order.



   Notice that the graph of $y = \frac{1}{2}x^2$ is wider than the graph of the parent function. The graphs of $y = 2x^2$ and $y = 3x^2$ are narrower than the graph of the parent function.

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

CC.9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k … and illustrate an explanation of the effects on the graph using technology. **CC.9-12.F.IF.7***

Try This

1. How would the graph of $y = 6x^2$ compare with the graph of the parent function? much narrower
2. How would the graph of $y = \frac{1}{5}x^2$ compare with the graph of the parent function? much wider
3. **Make a Conjecture** Make a conjecture about the effect of a on the graph of $y = ax^2$.

Consider the graphs of $y = -\frac{1}{2}x^2$, $y = -x^2$, $y = -2x^2$, and $y = -3x^2$.

4. Describe the differences in the graphs.
5. How would the graph of $y = -8x^2$ compare with the graph of $y = -x^2$? much narrower
6. How do these results affect your conjecture from Problem 3?
   When a is negative, the parabolas open downward. Otherwise the conjecture is the same.

Consider the graphs of $y = x^2 - 1$, $y = x^2$, $y = x^2 + 2$, and $y = x^2 + 4$.

7. Describe the differences in the graphs.
8. How would the graph of $y = x^2 - 7$ compare with the graph of the parent function? lower, with a vertex at $(0, -7)$
9. **Make a Conjecture** Make a conjecture about the effect of c on the graph of $y = x^2 + c$.
   The value of c causes the parabola to move up if $c > 0$ and down if $c < 0$.

Answers to *Try This*

3. The closer the value of a to 0, the wider the parabola. The greater the value of a, the narrower the parabola.
4. All the graphs open downward. The graph of $y = -\frac{1}{2}x^2$ is the widest. The graphs of $y = -2x^2$ and $y = -3x^2$ are narrower than the parent function.
7. The graph of $y = x^2 - 1$ is lower than the parent function. The graphs of $y = x^2 + 2$ and $y = x^2 + 4$ are higher.

COMMON CORE

8-4 Transforming Quadratic Functions

CC.9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); …. *Also* **CC.9-12.F.IF.7*, CC.9-12.F.IF.5***

Objective
Graph and transform quadratic functions.

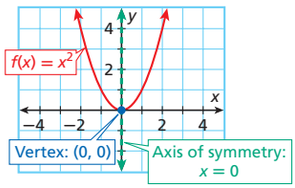
Why learn this?
You can compare how long it takes raindrops to reach the ground from different heights. (See Exercise 18.)

Remember!
Recall that the graphs of all linear functions are transformations of the linear parent function, $f(x) = x$.

The quadratic parent function is $f(x) = x^2$. The graph of all other quadratic functions are transformations of the graph of $f(x) = x^2$.

For the parent function $f(x) = x^2$:

- The axis of symmetry is $x = 0$, or the y-axis.
- The vertex is $(0, 0)$.
- The function has only one zero, 0.



Compare the coefficients in the following functions.

$f(x) = x^2 \quad g(x) = \frac{1}{2}x^2$

$h(x) = -3x^2$

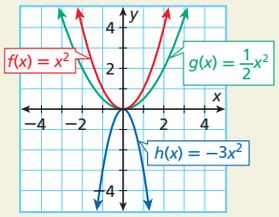
$f(x) = 1x^2 + 0x + 0$

$g(x) = \frac{1}{2}x^2 + 0x + 0$

$h(x) = -3x^2 + 0x + 0$

Same	Different
• $b = 0$ • $c = 0$	• Value of a

Compare the graphs of the same functions.



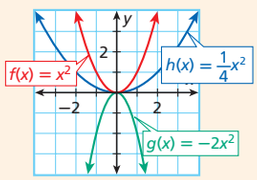
Same	Different
• Axis of symmetry is $x = 0$. • Vertex is $(0, 0)$.	• Widths of parabolas

The value of a in a quadratic function determines not only the direction a parabola opens, but also the width of the parabola.

Know it! Note

Width of a Parabola

WORDS	EXAMPLES
The graph of $f(x) = ax^2$ is **narrower** than the graph of $f(x) = x^2$ if $\lvert a\rvert > 1$ and **wider** if $\lvert a\rvert < 1$.	Compare the graphs of $g(x)$ and $h(x)$ with the graph of $f(x)$. $\lvert -2\rvert \stackrel{?}{>} 1$ $\quad \lvert \frac{1}{4}\rvert \stackrel{?}{<} 1$ $2 > 1$ $\quad \frac{1}{4} < 1$ narrower $\quad$ wider



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1 Introduce

EXPLORATION

Transforming Quadratic Functions

Look for patterns as you use a graphing calculator to graph quadratic functions.

1. Enter $y = x^2$ in your calculator as **Y1**. Then graph the function.
2. For each quadratic function in the table, enter the function as **Y2** and then graph the function in the same window as the graph of $y = x^2$. Find the vertex of the function and describe how the graph of the function compares to the graph of $y = x^2$.

Function	Vertex	How does the graph compare to the graph of $y = x^2$?
$y = x^2 + 2$	(0, 2)	Translated 2 units up
$y = x^2 + 5$	(0, 5)	Translated 5 units up
$y = x^2 - 6$	(0, −6)	Translated 6 units down
$y = x^2 - 10$	(0, −10)	Translated 10 units down

3. Explain how the value of c determines how the graph of $y = x^2 + c$ is related to the graph of $y = x^2$. The graph of $y = x^2$ is translated c units up when $c > 0$ and $|c|$ units down when $c < 0$.

THINK AND DISCUSS

4. **Explain** how you can sketch the graph of $y = x^2 + 11$ without using your calculator. Graph $y = x^2$ but translate it 11 units up.

Motivate

Have students graph $f(x) = x^2$, $g(x) = x^2 + 3$, and $h(x) = x^2 - 2$ on the same coordinate plane. Ask students to name one similarity among the graphs. Possible answers: all open upward, same axis of symmetry, same shape

Ask students to name one difference among the graphs. Possible answer: they intersect the y-axis in different places.

Tell students that in this lesson they will learn to determine the effects of many parameter changes on the graph of a quadratic function.

Explorations and answers are provided in your online resources.

8-4 Organizer

Pacing: Traditional $1\frac{1}{2}$ days
Block $\frac{3}{4}$ day

Objective: Graph and transform quadratic functions.

Algebra Lab
In *Algebra Lab Activities*

Technology Lab
In *Technology Lab Activities*

Online Edition
Tutorial Videos, Interactivity

Countdown

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Warm Up

For each quadratic function, find the axis of symmetry and vertex, and state whether the function opens upward or downward.

1. $y = x^2 + 3$ $x = 0$; $(0, 3)$; upward
2. $y = 2x^2$ $x = 0$; $(0, 0)$; upward
3. $y = -0.5x^2 - 4$ $x = 0$; $(0, -4)$; downward

Also available online

Math Humor

May I help you?

Yes, I would like to order $f(x) = 4x^2$ and $g(x) = 3x^2$.

What are you talking about?

My teacher told me to order my quadratic functions.

COMMON CORE

CC.9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Also* **CC.9-12.F.IF.7*, CC.9-12.F.IF.5***

Learn It Online
State Resources Online

Teaching Tip **Inclusion** When ordering the functions in **Example 1,** have students write the absolute value of a for each function, order these values from greatest to least, and then write the functions in order from narrowest to widest.

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Additional Examples

Example 1

Order the functions from narrowest graph to widest.

A. $f(x) = 3x^2, g(x) = 0.5x^2$ $f(x), g(x)$

B. $f(x) = x^2, g(x) = \frac{1}{2}x^2, h(x) = -2x^2$ $h(x), f(x), g(x)$

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 1

- What two properties of the graph of $y = ax^2$ can be determined from the value of a?

Teaching Tip **Visual** Students may not think the parabolas at the bottom of this page look like they have the same width. Have them compare the widths at specific locations above the vertex. For example, at 1 unit above the vertex, each parabola is 2 units wide.

EXAMPLE 1 Comparing Widths of Parabolas

Order the functions from narrowest graph to widest.

A $f(x) = -2x^2, g(x) = \frac{1}{3}x^2, h(x) = 4x^2$

Step 1 Find $|a|$ for each function.

$|-2| = 2 \quad \left|\frac{1}{3}\right| = \frac{1}{3} \quad |4| = 4$

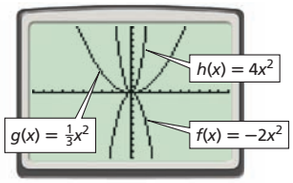
Step 2 Order the functions.

$h(x) = 4x^2$
$f(x) = -2x^2$
$g(x) = \frac{1}{3}x^2$

The function with the narrowest graph has the greatest value for $|a|$.

Check Use a graphing calculator to compare the graphs.

$h(x) = 4x^2$ has the narrowest graph, and $g(x) = \frac{1}{3}x^2$ has the widest graph. ✓



B $f(x) = 2x^2, g(x) = -2x^2$

Step 1 Find $|a|$ for each function.

$|2| = 2 \quad |-2| = 2$

Step 2 Order the functions from narrowest graph to widest.
Since the absolute values are equal, the graphs are the same width.

CHECK IT OUT! **Order the functions from narrowest graph to widest.**

1a. $f(x) = -x^2, g(x) = \frac{2}{3}x^2$ $f(x), g(x)$

1b. $f(x) = -4x^2, g(x) = 6x^2, h(x) = 0.2x^2$ $g(x), f(x), h(x)$

Compare the coefficients in the following functions.

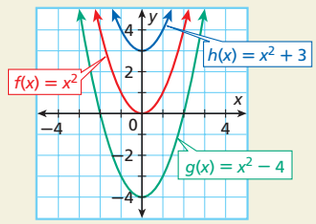
$f(x) = x^2 \quad g(x) = x^2 - 4$

$h(x) = x^2 + 3$

$f(x) = 1x^2 + 0x + 0$
$g(x) = 1x^2 + 0x + (-4)$
$h(x) = 1x^2 + 0x + 3$

Same	Different
• $a = 1$ • $b = 0$	• Value of c

Compare the graphs of the same functions.



Same	Different
• Axis of symmetry is $x = 0$. • Width of parabola	• Vertex of parabola

The value of c makes these graphs look different. The value of c in a quadratic function determines not only the value of the y-intercept but also a vertical translation of the graph of $f(x) = ax^2$ up or down the y-axis.

2 Teach

Guided Instruction

Review how to find the axis of symmetry and vertex of a parabola. Introduce the quadratic parent function, $f(x) = x^2$. Remind students that they have already studied transformations of linear functions from the parent function $f(x) = x$. Encourage students to compare the effects of each parameter change on the graphs of quadratic functions to the effects of the corresponding parameter change on the graphs of linear functions.

Reaching All Learners

Through Visual Cues

Have students create a design made of transformed parabolas and keep a record of each parabola's function. Encourage students to use their imagination with colors and patterns. Allow students to use graphing calculators to check their work. Some students will be motivated to explore additional types of transformations.

Vertical Translations of a Parabola

The graph of the function $f(x) = x^2 + c$ is the graph of $f(x) = x^2$ translated vertically.

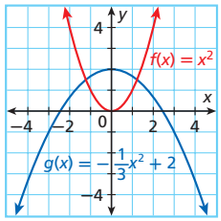
- If $c > 0$, the graph of $f(x) = x^2$ is translated c units **up**.
- If $c < 0$, the graph of $f(x) = x^2$ is translated c units **down**.

EXAMPLE 2 Comparing Graphs of Quadratic Functions

Compare the graph of each function with the graph of $f(x) = x^2$.

A $g(x) = -\frac{1}{3}x^2 + 2$

Method 1 Compare the graphs.



- The graph of $g(x) = -\frac{1}{3}x^2 + 2$ is **wider** than the graph of $f(x) = x^2$.
- The graph of $g(x) = -\frac{1}{3}x^2 + 2$ opens **downward**, and the graph of $f(x) = x^2$ opens **upward**.
- The axis of symmetry is the same.
- The vertex of $f(x) = x^2$ is $(0, 0)$. The vertex of $g(x) = -\frac{1}{3}x^2 + 2$ is translated **2 units up** to $(0, 2)$.

Helpful Hint

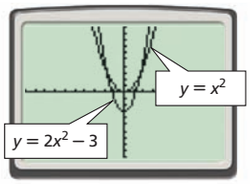
When comparing graphs, it is helpful to draw them on the same coordinate plane.

B $g(x) = 2x^2 - 3$

Method 2 Use the functions.

- Since $|2| > |1|$, the graph of $g(x) = 2x^2 - 3$ is **narrower** than the graph of $f(x) = x^2$.
- Since $-\frac{b}{2a} = 0$ for both functions, the axis of symmetry is the same.
- The vertex of $f(x) = x^2$ is $(0, 0)$. The vertex of $g(x) = 2x^2 - 3$ is translated **3 units down** to $(0, -3)$.

Check Use a graph to verify all comparisons.



2a. The graph of $g(x)$ is the same width, has the same axis of symmetry, opens downward, and its vertex is translated 4 units down to $(0, -4)$.

2b. The graph of $g(x)$ is narrower, has the same axis of symmetry, and also opens upward, and its vertex is translated 9 units up to $(0, 9)$.

2c. The graph of $g(x)$ is wider, has the same axis of symmetry, and also opens upward, and its vertex is translated 2 units up to $(0, 2)$.

CHECK IT OUT! **Compare the graph of each function with the graph of $f(x) = x^2$.**

2a. $g(x) = -x^2 - 4$ **2b.** $g(x) = 3x^2 + 9$ **2c.** $g(x) = \frac{1}{2}x^2 + 2$

The quadratic function $h(t) = -16t^2 + c$ can be used to approximate the height h in feet above the ground of a falling object t seconds after it is dropped from a height of c feet. This model is used only to approximate the height of falling objects because it does not account for air resistance, wind, and other real-world factors.

Teaching Tip **Inclusion** To help students remember how a affects the width of a parabola, compare $|a|$ to $|m|$, where m is the slope of a line. As $|a|$ increases, the parabola narrows (gets steeper), just as when $|m|$ increases, the line gets steeper. For example, $y = 10x^2$ is a very narrow parabola and $y = 10x$ is a very steep line. Also, $y = \frac{1}{10}x^2$ is a very wide parabola and $y = \frac{1}{10}x$ is an almost horizontal line.

COMMON ERROR ALERT

Some students mistakenly associate a greater absolute value of a with a wider parabola. Encourage students to associate the absolute value of a with the rate at which the graph rises or falls. The greater the absolute value of a, the faster the graph rises or falls and the narrower it is.

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Additional Examples

Example 2

Compare the graph of each function with the graph of $f(x) = x^2$.

A. $g(x) = -\frac{1}{4}x^2 + 3$

- The graph of $g(x)$ is wider than the graph of $f(x)$.
- The graph of $g(x)$ opens downward, and the graph of $f(x)$ opens upward.
- Both have the axis of symmetry $x = 0$.
- The vertex of $g(x)$ is $(0, 3)$, and the vertex of $f(x)$ is $(0, 0)$.

B. $g(x) = 3x^2$

- The graph of $g(x)$ is narrower than the graph of $f(x)$.
- Both graphs open upward.
- Both have the axis of symmetry $x = 0$.
- Both have the vertex at $(0, 0)$.

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 2

- What does each term in the function rule tell you about the graph?
- Does shifting the graph up or down change the axis of symmetry?
- What shifts the graph up or down?

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Additional Examples

Example 3

Two identical softballs are dropped. The first is dropped from a height of 400 feet and the second is dropped from a height of 324 feet.

a. Write the two height functions and compare their graphs. The graph of $h_1(t) = -16t^2 + 400$ is a vertical translation of the graph of $h_2(t) = -16t^2 + 324$; the y-intercept of h_1 is 76 units higher than that of h_2.

b. Use the graphs to tell when each softball reaches the ground. 5 s from 400 ft; 4.5 s from 324 ft

Also available online

Questioning Strategies

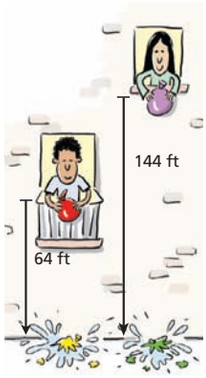
EXAMPLE 3

- What are reasonable domain and range values for these functions?
- How can you find the number of units that the graph of h_2 was translated from the graph of h_1?

Technology Have students set **Ymin** = 0 and **Xmin** = 0 when graphing **Example 3** on a graphing calculator. **Ymax** should be set to the greater height.

EXAMPLE 3 ***Physics Application***

Two identical water balloons are dropped from different heights as shown in the diagram.



a. Write the two height functions and compare their graphs.

Step 1 Write the height functions. The y-intercept c represents the original height.

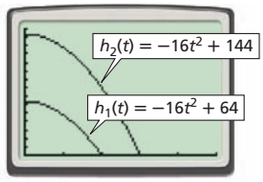
$h_1(t) = -16t^2 + 64$ *Dropped from 64 feet*

$h_2(t) = -16t^2 + 144$ *Dropped from 144 feet*

Caution!

Remember that the graphs shown here represent the *height* of the objects over time, not the *paths* of the objects.

Step 2 Use a graphing calculator. Since time and height cannot be negative, set the window for nonnegative values.



The graph of h_2 is a vertical translation of the graph of h_1. Since the balloon in h_2 is dropped from 80 feet higher than the one in h_1, the y-intercept of h_2 is 80 units higher.

b. Use the graphs to tell when each water balloon reaches the ground.

The zeros of each function are when the water balloons reach the ground.

The water balloon dropped from 64 feet reaches the ground in 2 seconds. The water balloon dropped from 144 feet reaches the ground in 3 seconds.

Check These answers seem reasonable because the water balloon dropped from a greater height should take longer to reach the ground.

3a. $h_1(t) = -16t^2 + 16$, $h_2(t) = -16t^2 + 100$; The graph of the ball that is dropped from a height of 100 ft is a vertical translation of the graph of the ball that is dropped from a height of 16 ft. The y-intercept of the graph of the ball that is dropped from 100 ft is 84 units higher.

3b. The ball that is dropped from 16 ft reaches the ground in 1 s. The ball that is dropped from 100 ft reaches the ground in 2.5 s.

CHECK IT OUT!

3. Two tennis balls are dropped, one from a height of 16 feet and the other from a height of 100 feet.

a. Write the two height functions and compare their graphs.

b. Use the graphs to tell when each tennis ball reaches the ground.

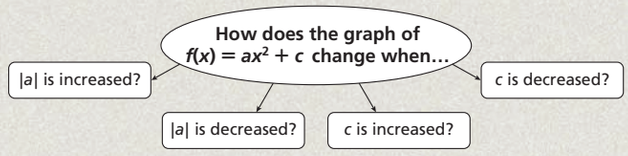
MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Describe how the graph of $f(x) = x^2 + c$ differs from the graph of $f(x) = x^2$ when the value of c is positive and when the value of c is negative.

2. Tell how to determine whether a graph of a function is wider or narrower than the graph of $f(x) = x^2$.

3. GET ORGANIZED Copy and complete the graphic organizer by explaining how each change affects the graph $f(x) = ax^2 + c$.



Know it! Note

3 Close

Summarize

Have students write a function to describe each of the following situations.

The graph of $f(x) = x^2$ is

- translated 3 units down. $g(x) = x^2 - 3$
- narrowed. Possible answer: $g(x) = 5x^2$
- widened and translated 2 units up. Possible answer: $g(x) = 0.25x^2 + 2$
- the same width and location but opens downward. $g(x) = -x^2$

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. When c is positive, the graph of $y = x^2 + c$ is a translation c units up of the graph of $y = x^2$. When c is negative, the graph of $y = x^2 + c$ is a translation c units down of the graph of $y = x^2$.
2. The graph of $g(x) = ax^2$ is narrower than the graph of $f(x) = x^2$ if $|a| > 1$, and wider if $|a| < 1$.
3. See Additional Answers.

8-4 Exercises

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GUIDED PRACTICE

SEE EXAMPLE 1

Order the functions from narrowest graph to widest.

1. $f(x) = 3x^2, g(x) = 2x^2$ **f(x), g(x)**
2. $f(x) = 5x^2, g(x) = -5x^2$ **The graphs have the same width.**
3. $f(x) = \frac{3}{4}x^2, g(x) = -2x^2, h(x) = -8x^2$ **h(x), g(x), f(x)**
4. $f(x) = x^2, g(x) = -\frac{4}{5}x^2, h(x) = 3x^2$ **h(x), f(x), g(x)**

SEE EXAMPLE 2

Compare the graph of each function with the graph of $f(x) = x^2$.

5. $g(x) = x^2 + 6$
6. $g(x) = -2x^2 + 5$
7. $g(x) = \frac{1}{3}x^2$
8. $g(x) = -\frac{1}{4}x^2 - 2$

SEE EXAMPLE 3

9. **Multi-Step** Two baseballs are dropped, one from a height of 16 feet and the other from a height of 256 feet.
   a. Write the two height functions and compare their graphs.
   b. Use the graphs to tell when each baseball reaches the ground.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
10–13	1
14–17	2
18	3

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Order the functions from narrowest graph to widest.

10. $f(x) = x^2, g(x) = 4x^2$ **g(x), f(x)**
11. $f(x) = -2x^2, g(x) = \frac{1}{2}x^2$ **f(x), g(x)**
12. $f(x) = -x^2, g(x) = -\frac{5}{8}x^2, h(x) = \frac{1}{2}x^2$ **f(x), g(x), h(x)**
13. $f(x) = -5x^2, g(x) = -\frac{3}{8}x^2, h(x) = 3x^2$ **f(x), h(x), g(x)**

Compare the graph of each function with the graph of $f(x) = x^2$.

14. $g(x) = \frac{1}{2}x^2 - 10$
15. $g(x) = -4x^2 - 2$
16. $g(x) = \frac{2}{3}x^2 - 9$
17. $g(x) = -\frac{1}{5}x^2 + 1$
18. **Multi-Step** A raindrop falls from a cloud at an altitude of 10,000 ft. Another raindrop falls from a cloud at an altitude of 14,400 ft.
    a. Write the two height functions and compare their graphs.
    b. Use the graphs to tell when each raindrop reaches the ground.

18a. $h_1(t) = -16t^2 + 10{,}000$, $h_2(t) = -16t^2 + 14{,}400$; The graph of h_2 is a vertical translation of the graph of h_1. The y-intercept of h_2 is 4400 units higher.

Tell whether each statement is sometimes, always, or never true.

19. The graphs of $f(x) = ax^2$ and $g(x) = -ax^2$ have the same width. **always**
20. The function $f(x) = ax^2 + c$ has three zeros. **never**
21. The graph of $y = ax^2 + 1$ has its vertex at the origin. **never**
22. The graph of $y = -x^2 + c$ intersects the x-axis. **sometimes**
23. **Data Collection** Use a graphing calculator and a motion detector to graph the height of a falling object over time. **Check students' work.**
    a. Find a function to model the height of the object while it is in motion.
    b. **Critical Thinking** Explain why the value of a in your function is not -16.

23b. Possible answer: Air resistance or wind may have affected the fall of the object.

8-4 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–3**
Basic 10–32, 35, 34, 38–40
Average 10–42
Advanced 10–43

Homework Quick Check
Quickly check key concepts.
Exercises: 12, 16, 18, 35

Data Collection To help students complete **Exercise 23,** see *Technology Lab Activities*.

Answers

5. The graph of $g(x)$ is the same width, has the same axis of symmetry, and also opens upward; its vertex is translated 6 units up to $(0, 6)$.
6. The graph of $g(x)$ is narrower, has the same axis of symmetry, and opens downward; its vertex is translated 5 units up to $(0, 5)$.
7. The graph of $g(x)$ is wider, has the same axis of symmetry, also opens upward, and has the same vertex.
8. The graph of $g(x)$ is wider, has the same axis of symmetry, and opens downward; its vertex is translated 2 units down to $(0, -2)$.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 9, 18, 34, 38

Reason abstractly and quantitatively. Exercises 19–22

Construct viable arguments and critique the reasoning of others. Exercises 33, 36–37

Use appropriate tools strategically. Exercises 23, 42

Look for and make use of structure. Exercise 35

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9a. $h_1(t) = -16t^2 + 16$, $h_2(t) = -16t^2 + 256$; the graph of h_2 is a vertical translation of the graph of h_1. The y-intercept of the graph h_2 is 240 units higher.

b. The baseball that is dropped from 256 ft reaches the ground in 4 s. The baseball that is dropped from 16 ft reaches the ground in 1 s.

14. The graph of $g(x)$ is wider, has the same axis of symmetry, and also opens upward; its vertex is translated 10 units down to $(0, -10)$.

15. The graph of $g(x)$ is narrower, has the same axis of symmetry, and opens downward; its vertex is translated 2 units down to $(0, -2)$.

16. The graph of $g(x)$ is wider, has the same axis of symmetry, also opens upward, and its vertex is translated 9 units down to $(0, -9)$.

17. The graph of $g(x)$ is wider, has the same axis of symmetry, and opens downward; its vertex is translated 1 unit up to $(0, 1)$.

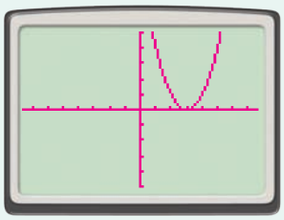
18b. 10,000 ft: 25 s; 14,400 ft: 30 s

MULTI-STEP TEST PREP **Exercise 38** involves finding the height of an object as a function of time. This exercise prepares students for the Multi-Step Test Prep.

Answers

37. The graph of $f(x) = \frac{1}{10}x^2 - 5$ will look like the graph of $f(x) = x^2$, except that it will be wider (because $\frac{1}{10} < 1$) and it will be translated 5 units down (because $c = -5$).

38a.



$y = (x - 3)^2$ and $y = x^2 - 3$ have the same width as $y = x^2$, and all of these parabolas open upward. $y = (x - 3)^2$ is translated 3 units right of $y = x^2$, has an axis of symmetry of $x = 3$, and has 1 zero (like $y = x^2$). $y = x^2 - 3$ has 2 zeros and is translated 3 units down from $y = x^2$.

b. 64 ft; 4 s

c. The x-coordinate of the vertex is the amount subtracted from x in the quantity that is squared, and the y-coordinate is the constant term in the equation. On the graph, find the greatest value of y, and then find the x-coordinate that corresponds to this y-coordinate.

27. Possible answer: $f(x) = -10x^2 + 2$

28. Possible answer: $f(x) = \frac{1}{2}x^2 + 1$

34a. 50 ft and 75 ft

b. $h(t) = -16t^2 + 75$

Write a function to describe each of the following.

24. The graph of $f(x) = x^2 + 10$ is translated 10 units down. $f(x) = x^2$

25. The graph of $f(x) = 3x^2 - 2$ is translated 4 units down. $f(x) = 3x^2 - 6$

26. The graph of $f(x) = 0.5x^2$ is narrowed. **Possible answer:** $f(x) = x^2$

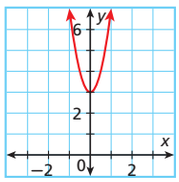
27. The graph of $f(x) = -5x^2$ is narrowed and translated 2 units up.

28. The graph of $f(x) = x^2 - 7$ is widened and has no x-intercept.

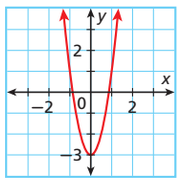
Match each function to its graph.

29. $f(x) = 4x^2 - 3$ **B** **30.** $f(x) = \frac{1}{4}x^2 - 3$ **D** **31.** $f(x) = 4x^2 + 3$ **A** **32.** $f(x) = -\frac{1}{4}x^2 - 3$ **C**

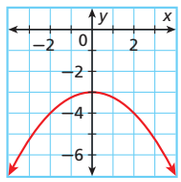
A.



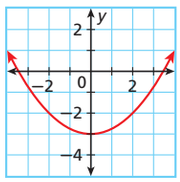
B.



C.



D.



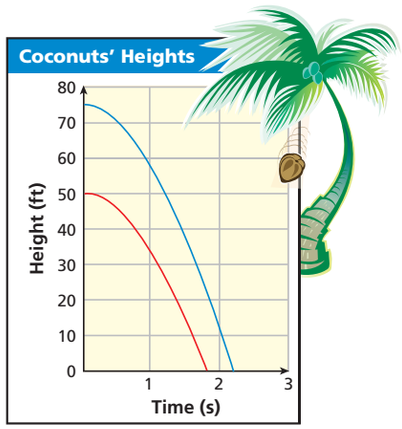
33. Critical Thinking For what values of a and c will $f(x) = ax^2 + c$ have one zero? $c = 0$ and $a \neq 0$

34. Physics The graph compares the heights of two identical coconuts that fell from different trees.

a. What are the starting heights of each coconut?

b. What is a possible function for the blue graph?

c. Estimate the time for each coconut to reach the ground. **about 1.8 s and 2.2 s**



35. Possible answers:

a. $f(x) = x^2 + 2$

b. $f(x) = -x^2 + 2$

c. $f(x) = 5x^2 + 2$

35. Give an example of a quadratic function for each description.

a. Its graph opens upward.

b. Its graph has the same width as in part **a**, but the graph opens downward.

c. Its graph is narrower than the graph in part **a**.

36. c **is the** y**-intercept of** $y = x^2 + c$**, and** b **is the** y**-intercept of** $y = x + b$**.**

36. Critical Thinking Describe how the effect that the value of c has on the graph of $y = x^2 + c$ is similar to the effect that the value of b has on the graph of $y = x + b$.

37. Write About It Explain how you know, without graphing, what the graph of $f(x) = \frac{1}{10}x^2 - 5$ looks like.

38. a. Use a graphing calculator to graph $y = (x - 3)^2$. Compare this graph to the graph of $y = x^2$. How does this differ from $y = x^2 - 3$?

b. The equation $h = -16(x - 2)^2 + 64$ describes the height in feet of a water bottle rocket as a function of time. What is the highest point that the rocket will reach? When will it return to the ground?

c. How can the vertex be located from the equation? from the graph?

Sam Dudgeon/HMH

PRACTICE A

PRACTICE C

PRACTICE B

Name _____ Date _____ Class _____

LESSON 8-4 **Practice B**

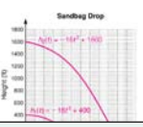
Transforming Quadratic Functions

Order the functions from narrowest graph to widest.

1. $f(x) = 3x^2$; $g(x) = -2x^2$
   $f(x)$, $g(x)$
2. $f(x) = \frac{1}{2}x^2$; $g(x) = 5x^2$; $h(x) = x^2$
   $g(x)$, $h(x)$, $f(x)$
3. $f(x) = 4x^2$; $g(x) = -3x^2$; $h(x) = \frac{1}{4}x^2$
   $f(x)$, $g(x)$, $h(x)$
4. $f(x) = 0.5x^2$; $g(x) = \frac{1}{4}x^2$; $h(x) = \frac{1}{3}x^2$
   $f(x)$, $h(x)$, $g(x)$

Compare the graph of each function with the graph of $f(x) = x^2$.

5. $g(x) = 5x^2 + 10$ The graph of $g(x)$ is narrower. Both graphs open upward. The vertex of $g(x)$, (0, 10), is translated 10 units up from the vertex of $f(x)$ at (0, 0).
6. $g(x) = \frac{1}{8}x^2 - 3$ The graph of $g(x)$ is wider. Both graphs open upward. The vertex of $g(x)$, (0, –3), is translated 3 units down from the vertex of $f(x)$ at (0, 0).
7. $g(x) = -3x^2 + 8$ The graph of $g(x)$ is narrower. $g(x)$ opens downward and $f(x)$ opens upward. The vertex of $g(x)$, (0, 8), is translated 8 units up from the vertex of $f(x)$ at (0, 0).
8. $g(x) = -\frac{3}{4}x^2 + \frac{1}{4}$ The graph of $g(x)$ is wider. $g(x)$ opens downward and $f(x)$ opens upward. The vertex of $g(x)$, $(0, \frac{1}{4})$, is translated $\frac{1}{4}$ units up from the vertex of $f(x)$ at (0, 0).
9. Two sandbags are dropped from a hot air balloon, one from a height of 400 feet and the other from a height of 1600 feet.
   a. Write the two height functions.
      $h_1(t) = -16t^2 + 400$ $h_2(t) = -16t^2 + 1600$
   b. Sketch and compare their graphs.
      The graph of h_2 is a vertical translation of the graph of h_1: 1200 units up.



READING STRATEGIES

Name _____ Date _____ Class _____

LESSON 8-4 **Reading Strategies**

Identify Relationships

The table shows how changing the values of a and c affect the shape and location of a parabola. All the functions are in the form $f(x) = ax^2 + c$.

$f(x) = x^2$ $g(x) = 4x^2$	$f(x) = x^2$ $g(x) = \frac{1}{2}x^2$	$f(x) = x^2$ $g(x) = x^2 + 5$	$f(x) = x^2$ $g(x) = x^2 - 7$
When $\lvert a \rvert$ increases, the parabola becomes narrower.	When $\lvert a \rvert$ decreases, the parabola becomes wider.	When c increases, the parabola is translated up.	When c decreases, the parabola is translated down.

Use the functions above to answer the following questions.

1. Which is wider: $g(x) = 4x^2$ or $g(x) = \frac{1}{2}x^2$? $g(x) = \frac{1}{2}x^2$
   How would you be able to tell without looking at their graphs?
   The absolute value of $\frac{1}{2}$ is less than the absolute value of 4.
2. Why would $g(x) = 4x^2$ and $h(x) = -4x^2$ have the same width?
   4 and –4 have the same absolute value: 4.
   How would their graphs be different? $g(x)$ will open upward, $h(x)$ will open downward.
3. Order the graphs of the following functions from narrowest (1) to widest (4) by numbering.
   $f(x) = -2x^2$ 2 $g(x) = \frac{1}{4}x^2$ 4 $h(x) = 3x^2$ 1 $j(x) = -x^2$ 3
4. Which has its vertex below the x-axis: $g(x) = x^2 + 5$ or $g(x) = x^2 - 7$? $g(x) = x^2 - 7$
   How would you be able to tell without looking at their graphs?
   c is negative, so it was translated down from the origin.
5. How many vertical units separate the y-intercepts of $g(x) = x^2 + 5$ or $g(x) = x^2 - 7$?

RETEACH

Name _____ Date _____ Class _____

LESSON 8-4 **Review for Mastery**

Transforming Quadratic Functions

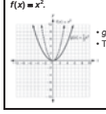
Compared to the function $f(x) = x^2$, a quadratic function will become narrower or wider depending on the value of a. It will translate up or down depending on the value of c.

For a quadratic function $f(x) = ax^2 + bx + c$:

If $\lvert a \rvert < 1$	graph is wider
If $\lvert a \rvert > 1$	graph is narrower
For any change in a, the vertex and axis of symmetry are the same.	

If $c > 0$	graph shifts up
If $c < 0$	graph shifts down
For any change in c, the vertex changes. The axis of symmetry is the same.	

Compare the graph of $g(x) = \frac{1}{3}x^2$ to $f(x) = x^2$.



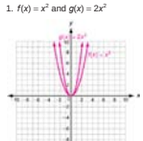
- $g(x)$ is wider than $f(x)$.
- The vertex is the same.

Compare the graph of $h(x) = x^2 - 4$ to $f(x) = x^2$.

- $h(x)$ is translated down 4 units.
- The vertex of $f(x)$ is (0, 0). The vertex of $g(x)$ is (0, –4).

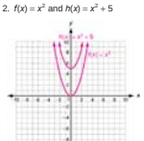
Compare the graphs of the functions below.

1. $f(x) = x^2$ and $g(x) = 2x^2$



The graph of $g(x)$ is narrower than the graph of $f(x)$.

2. $f(x) = x^2$ and $h(x) = x^2 + 5$



The graph of $h(x)$ is translated up 5 units from the graph of $f(x)$.

TEST PREP

39. Which function's graph is the result of shifting the graph of $f(x) = -x^2 - 4$ 3 units down?

Ⓐ $g(x) = -x^2 - 1$ Ⓒ $g(x) = -4x^2 - 4$

Ⓑ $g(x) = -\frac{1}{3}x^2 - 4$ Ⓓ $g(x) = -x^2 - 7$

40. Which of the following is true when the graph of $f(x) = x^2 + 4$ is transformed into the graph of $g(x) = 2x^2 + 4$?

Ⓕ The new function has more zeroes than the old function.

Ⓖ Both functions have the same vertex.

Ⓗ The function is translated up.

Ⓙ The axis of symmetry changes.

41. Gridded Response For what value of c will $f(x) = x^2 + c$ have one zero? **0**

CHALLENGE AND EXTEND

42. Graphing Calculator Graph the functions $f(x) = (x + 1)^2$, $g(x) = (x + 4)^2$, $h(x) = (x - 2)^2$, and $k(x) = (x - 5)^2$. Make a conjecture about the result of transforming the graph of $f(x) = x^2$ into the graph of $f(x) = (x - h)^2$.

42. The graph of $f(x) = (x - h)^2$ is the result of translating the graph of $f(x) = x^2$ by h units along the x-axis.

43. Using the function $f(x) = x^2$, write each new function:

a. The graph is translated 7 units down. $f(x) = x^2 - 7$

b. The graph is reflected across the x-axis and translated 2 units up. $f(x) = -x^2 + 2$

c. Each y-value is halved, and then the graph is translated 1 unit up. $f(x) = \frac{1}{2}x^2 + 1$

PROBLEM SOLVING

Name _____ Date _____ Class _____

LESSON 8-4 **Problem Solving**
Transforming Quadratic Functions

Write the correct answer.

1. Two construction workers working at different heights on a skyscraper dropped their hammers at the same time. The first was working at a height of 400 ft, the second at a height of 160 ft. Write the two functions that describe the heights of the hammers.
   $y = -16x^2 + 400$
   $y = -16x^2 + 160$
2. Graph the two functions you found in problem 1 on the grid below.
3. Based on the graphs you drew in problem 2, how long will it take each hammer to reach the ground?
   3.2 seconds
   5 seconds

The pull of gravity varies from planet to planet. The graph shows the height of objects dropped from 500 ft on the surface of four planets. Use this graph to answer questions 4–6. Select the best answer.

4. Of the four planets, Jupiter has the strongest gravity. Which of the four graphs represents the height of the object dropped on Jupiter?
   F Graph 1 H Graph 3
   G Graph 2 J Graph 4
5. Which of the graphs represents an object dropped on Earth?
   A Graph 1 C Graph 3
   B Graph 2 D Graph 4
6. Due to its small size, Pluto has a very weak pull of gravity. Which of the equations below represents the graph of the object dropped on Pluto?
   A $h(t) = -41x^2 + 500$
   B $h(t) = -16x^2 + 500$
   C $h(t) = -6x^2 + 500$
   D $h(t) = -1.25x^2 + 500$

CHALLENGE

Name _____ Date _____ Class _____

LESSON 8-4 **Challenge**
Vertex Form

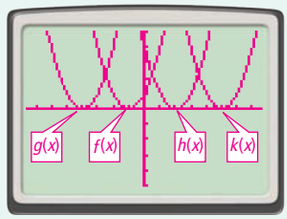
You've learned that any quadratic function can be written in the form $f(x) = ax^2 + bx + c$. This form is called **standard form.**
Any quadratic function can also be written in **vertex form.**
$f(x) = a(x - h)^2 + k$, where a, h, and k are real numbers and $a \neq 0$.

1. Use a graphing calculator to compare these functions.
   $f(x) = x^2$ $g(x) = (x - 3)^2$ $h(x) = (x - 6)^2$ $j(x) = (x + 4)^2$
   a. Which parameter, a, h, or k, changes in each function? h
   b. What is the value of that parameter in each function? 0, 3, 6, –4
   c. What characteristic of the graphs changes each time? vertex moves horizontally
   d. In terms of transformations, describe the effect of this parameter.
   The parameter h determines a horizontal translation.
2. Use a graphing calculator to compare these functions.
   $f(x) = x^2$ $g(x) = x^2 + 2$ $h(x) = x^2 + 5$ $j(x) = x^2 - 3$
   a. Which parameter changes in each function? k
   b. What is the value of that parameter in each function? 0, 2, 5, –3
   c. What characteristic of the graphs changes each time? vertex moves vertically
   d. In terms of transformations, describe the effect of this parameter.
   The parameter k determines a vertical translation.
3. Use a graphing calculator to compare these functions.
   $f(x) = (x - 3)^2 + 5$ $g(x) = 2(x - 3)^2 + 5$ $h(x) = -1(x - 3)^2 + 5$ $j(x) = \left(\frac{1}{5}\right)(x - 3)^2 + 5$
   a. Which parameter changes in each function? a
   b. What is the value of that parameter in each function? 1, 2, –1, $\frac{1}{5}$
   c. What characteristic of the graphs changes each time? width and direction
   d. In terms of transformations, describe the effect of this parameter.
   The parameter a makes the parabola narrower or wider and makes the parabola open up or down.
4. Why do you think $f(x) = a(x - h)^2 + k$ is called vertex form?
   The coordinates of the vertex are (h, k).

TEST PREP DOCTOR Shifting the quadratic function in **Exercise 39** will change only the value of c. Eliminate choices **B** and **C.**

Answers

42.



Journal

Describe the effects that a and c have on the graph of $y = ax^2 + c$.

ALTERNATIVE ASSESSMENT

Write a quadratic function that is wider than $f(x) = x^2$ and translated 3 units down. Write a second function that is narrower and translated 2 units up. Then compare each graph to $f(x) = x^2$.

Power Presentations with PowerPoint®

8-4 Lesson Quiz

1. Order the functions $f(x) = 4x^2$, $g(x) = -5x^2$, and $h(x) = 0.8x^2$ from narrowest graph to widest. $g(x)$, $f(x)$, $h(x)$

2. Compare the graph of $g(x) = 0.5x^2 - 2$ with the graph of $f(x) = x^2$.

- The graph of $g(x)$ is wider.
- Both graphs open upward.
- Both have the axis of symmetry $x = 0$.
- The vertex of $g(x)$ is $(0, -2)$; the vertex of $f(x)$ is $(0, 0)$.

Two identical soccer balls are dropped. The first is dropped from a height of 100 feet and the second is dropped from a height of 196 feet.

3. Write the two height functions and compare their graphs.
The graph of $h_1(t) = -16t^2 + 100$ is a vertical translation of the graph of $h_2(t) = -16t^2 + 196$; the y-intercept of h_1 is 96 units lower than that of h_2.

4. Use the graphs to tell when each soccer ball reaches the ground. 2.5 s from 100 ft; 3.5 from 196 ft

Also available online

SECTION 8A

MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and skills in a real-world format.

INTERACTIVE Online Edition

Answers

2. See Additional Answers.
4. The x-coordinate represents the time when the water bottle rocket reaches its maximum height, which is represented by the y-coordinate.
5. 1.775 s; for $y = 110$ the corresponding x-coordinate is 1.775.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

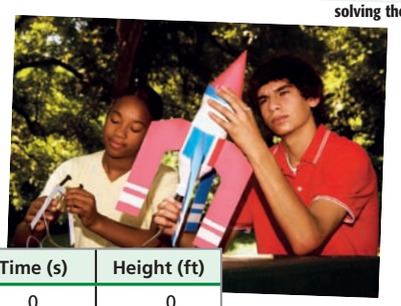
Learn It Online
State Resources Online

MULTI-STEP TEST PREP

MATHEMATICAL PRACTICES **Make sense of problems and persevere in solving them.**

Quadratic Functions

The Sky's the Limit The Physics Club is using computer simulation software to design a water bottle rocket that doesn't have a parachute. The data for their current design are shown in the table.



Time (s)	Height (ft)
0	0
1	80
2	128
3	144
4	128
5	80

1. Tell whether the data satisfy a quadratic function. yes
2. Graph the function from Problem 1.
3. Find and label the zeros, axis of symmetry, and vertex. 0 and 6; $x = 3$; (3,144)
4. Explain what the x- and y-coordinates of the vertex represent in the context of the problem.
5. Estimate how many seconds it will take the rocket to reach 110 feet. Explain.

(all) Sam Dudgeon/HMH

INTERVENTION

Scaffolding Questions

1. How can you tell from a table of data whether the data is linear or quadratic? linear: constant first differences quadratic: constant second differences
2. Do you need to graph negative x- or y-values? Explain. No; neither time nor height can be negative.
3. Which points do you look for when finding the zeros of a function from its graph? points that contain the x-intercepts
4. Does this graph have a maximum or a minimum? maximum What are the labels on the axes of your graph? vertical axis: Height (ft), horizontal axis: Time (s)
5. Between what two heights given in the table would 110 ft belong? between 80 ft and 128 ft What are the times associated with those two heights? 1 s and 2 s

Extension

If the function that models the height of a rocket is $h(t) = -16t^2 + 96t$, what equation would you solve to find the time it takes the rocket to reach 130 ft? How many answers are there? Explain. $130 = -16t^2 + 96t$; 2; there are 2 zeros.

READY TO GO ON?

CHAPTER 8

Quiz for Lessons 8-1 Through 8-4

8-1 Identifying Quadratic Functions

Tell whether each function is quadratic. Explain.

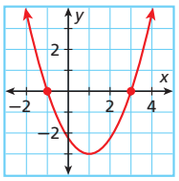
1. $y + 2x^2 = 3x$ — 1. Yes; the equation can be written in the form $y = ax^2 + bx + c$.
2. $x^2 + y = 4 + x^2$ — 2. No; $a = 0$
3. $\{(2, 12), (-1, 3), (0, 0), (1, 3)\}$ — 3. Yes; the second differences are constant.

Tell whether the graph of each quadratic function opens upward or downward and whether the parabola has a maximum or a minimum.

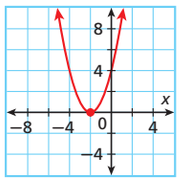
4. $y = -x^2 - 7x + 18$ — downward; maximum
5. $y - 2x^2 = 4x + 3$ — upward; minimum
6. $y = 5x - 0.5x^2$ — downward; maximum
7. Graph the function $y = \frac{1}{2}x^2 - 2$ and give the domain and range.

8-2 Characteristics of Quadratic Functions

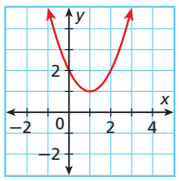
Find the zeros of each quadratic function from its graph. Then find the axis of symmetry.

8. 

−1 and 3; $x = 1$

9. 

−2; $x = -2$

10. 

no zeros; $x = 1$

Fnd the vertex.

11. $y = x^2 + 6x + 2$ $(-3, -7)$
12. $y = 3 + 4x - 2x^2$ $(1, 5)$
13. $y = 3x^2 + 12x - 12$ $(-2, -24)$
14. The height in feet of the curved roof of an aircraft hangar can be modeled by $y = -0.02x^2 + 1.6x$, where x is the horizontal distance in feet from one wall at ground level. What is the greatest height of the hangar? 32 ft

8-3 Graphing Quadratic Functions

Graph each quadratic function.

15. $y = x^2 + 3x + 9$
16. $y = x^2 - 2x - 15$
17. $y = x^2 - 2x - 8$
18. $y = 2x^2 - 6$
19. $y = 4x^2 + 8x - 2$
20. $y = 2x^2 + 10x + 1$

8-4 Transforming Quadratic Functions

Compare the graph of each function with the graph of $f(x) = x^2$.

21. $g(x) = x^2 - 2$
22. $g(x) = \frac{2}{3}x^2$
23. $g(x) = 5x^2 + 3$
24. $g(x) = -x^2 + 4$

25. The pilot of a hot-air balloon drops a sandbag onto a target from a height of 196 feet. Later, he drops an identical sandbag from a height of 676 feet.
    a. Write the two height functions and compare their graphs. Use $h(t) = -16t^2 + c$, where c is the height of the balloon. $h_1(t) = -16t^2 + 196$; $h_2(t) = -16t^2 + 676$
    b. Use the graphs to tell when each sandbag will reach the ground. 3.5 s; 6.5 s

SECTION 8A

READY TO GO ON?

Organizer

Objective: Assess students' mastery of concepts and skills in this section.

Resources

Assessment Resources
Section Quiz

INTERVENTION

Resources

***Ready to Go On? Intervention and Enrichment* Worksheets**
***Ready to Go On?* Online**

Answers

7, 15–24. See Additional Answers.

READY TO GO ON?

Diagnose and Prescribe

NO — INTERVENE

YES — ENRICH

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 8A: Worksheets	Online	
Lesson 8-1	8-1 Intervention	Activity 8-1	Diagnose and Prescribe Online
Lesson 8-2	8-2 Intervention	Activity 8-2	
Lesson 8-3	8-3 Intervention	Activity 8-3	
Lesson 8-4	8-4 Intervention	Activity 8-4	

***Ready to Go On? Enrichment*, Section 8A**
Worksheets
Online

SECTION 8B

Solving Quadratic Equations

One-Minute Section Planner

Lesson	Lab Resources	Materials
Lesson 8-5 Solving Quadratic Equations by Graphing • Solve quadratic equations by graphing. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Required** graphing calculator
Technology Lab Explore Roots, Zeros, and *x*-Intercepts • Use tables and graphs on a graphing calculator to explore the relationships between zeros, roots, and *x*-intercepts. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Technology Lab Activities*** Lab Recording Sheet	**Required** graphing calculator
Lesson 8-6 Solving Quadratic Equations by Factoring • Solve quadratic equations by factoring.		**Optional** graphing calculator
Lesson 8-7 Solving Quadratic Equations by Using Square Roots • Solve quadratic equations by factoring. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		
Algebra Lab Model Completing the Square • Use algebra tiles to model completing the square. ☐ SAT-10 ☑ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests	***Algebra Lab Activities*** Lab Recording Sheet	**Required** algebra tiles (MK)
Lesson 8-8 Completing the Square • Solve quadratic equations by completing the square. ☑ SAT-10 ☑ NAEP ☐ ACT ☑ SAT ☐ SAT Subject Tests		**Required** graphing calculator **Optional** algebra tiles (MK)
Lesson 8-9 The Quadratic Formula and the Discriminant • Solve quadratic equations by using the Quadratic Formula. • Determine the number of solutions of a quadratic equation by using the discriminant. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Optional** graphing calculator
Extension Cubic Functions and Equations • Recognize and graph cubic functions. • Solve cubic equations. ☐ SAT-10 ☐ NAEP ☐ ACT ☐ SAT ☑ SAT Subject Tests		
Lesson 8-10 Nonlinear Systems • Solve systems of equations in two variables in which one equation is linear and the other is quadratic. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Required** graph paper **Optional** graphing calculator

Note: If NAEP is checked, the content is tested on either the Grade 8 or the Grade 12 NAEP assessment.

MK = *Manipulatives Kit*

Selected Answers in Student Edition

The answers to most odd-numbered exercises are included in the Selected Answers section at the back of the Student Edition textbook.

All answers for the Check it Out! exercises in each lesson and for the Chapter Study Guide: Review are included in the Student Edition.

Math Background

COMMON CORE PROFESSIONAL DEVELOPMENT

QUADRATIC EQUATIONS

Lesson 8-5

A *quadratic equation* is an equation that can be written in the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$. This is the *standard form* of a quadratic equation. Every quadratic equation written in this form has a related quadratic function $y = ax^2 + bx + c$.

Just as with the solutions of all equations, a solution of a quadratic equation is a value of the variable that makes the equation true. One way to solve a quadratic equation in standard form is to graph the related quadratic function and find its x-intercepts.

For example, for the quadratic equation $x^2 + x - 6 = 0$, the graph of the related function $y = x^2 + x - 6$ is the set of all ordered pairs of the form $(x, x^2 + x - 6)$. It must be true that at any point where the graph intersects the x-axis, the y-coordinate is 0; that is, $x^2 + x - 6 = 0$. In other words, the x-intercepts of the graph are precisely the solutions of the quadratic equation.

Students should understand that this is a general method that works for any equation set equal to 0 whose related function can be graphed. For example, the linear equation $\frac{1}{2}x - 2 = 0$ may be solved by graphing the related linear function $y = \frac{1}{2}x - 2$ and noticing that the graph intersects the x-axis at $x = 4$.

SOLUTION METHODS

Lessons 8-5 to 8-9

Although graphing has the advantage of being a fairly intuitive solution method, it is often inefficient and imprecise. These drawbacks motivate the need for other solution techniques.

The method of solving a quadratic equation by factoring springs from the *Zero Product Property*: If the product of two real numbers is zero, then at least one of the numbers is zero. This property justifies solving quadratic equations by factoring. For example, to solve $x^2 + x - 6 = 0$, note that $x^2 + x - 6 = (x - 2)(x + 3)$. By the Zero Product Property, it must be true that $x - 2 = 0$ or $x + 3 = 0$, which gives the solutions 2 and -3.

Some quadratic equations do not lend themselves to factoring, so still more methods are needed. The techniques of completing the square and using the Quadratic Formula work for *any* quadratic equation. It is essential for students to realize that graphing, factoring, and the Quadratic Formula can be used to solve quadratic equations only when one side of the quadratic equation is equal to zero (i.e., the equation is written in standard form).

THE DISCRIMINANT

Lesson 8-9

Given a quadratic equation in standard form, the discriminant, $b^2 - 4ac$, offers an efficient means of determining the number of real solutions. If the discriminant is positive, there are two real solutions. If the discriminant is zero, there is one real solution. If the discriminant is negative, there are no real solutions.

According to the Fundamental Theorem of Algebra, every polynomial function of degree $n \geq 1$ has exactly n zeros. This means that all quadratic equations, whose related functions are polynomials of degree 2, have exactly two solutions. This may seem to contradict the previous discussion about the discriminant, but there are two important points.

If there is only one real solution, it has *multiplicity* 2. For example, the factored form of $x^2 - 2x + 1 = 0$ is $(x - 1)^2 = 0$, so $x = 1$ is the only solution. It is called a solution of multiplicity 2, meaning it is counted twice, since the factor $x - 1$ occurs twice. (An example of the analogous situation when dealing with numbers is $9 = 3^2$. Even though 3 is the only prime factor, we say that the prime factorization of 9 contains two factors.)

If there are no real solutions, the quadratic equation has two *complex* solutions. Complex numbers have the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. Students will study complex numbers in Algebra 2.

8-5 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Solve quadratic equations by graphing.

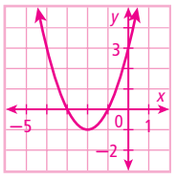
Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

1. Graph $y = x^2 + 4x + 3$.



2. Identify the vertex and zeros of the function above. vertex: $(-2, -1)$; zeros: $-3, -1$

Also available online

Math Humor

Q: What does a person have if they can easily find solutions to quadratic equations?

A: Zero visibility.

COMMON CORE

CC.9-12.A.REI.11 Explain why the x- coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values,….*

CC.9-12.A.REI.4 Solve quadratic equations in one variable.

Learn It Online
State Resources Online

COMMON CORE

8-5 Solving Quadratic Equations by Graphing

CC.9-12.A.REI.11 Explain why the x- coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately,….* *Also* **CC.9-12.A.REI.4, CC.9-12.F.IF.7*, CC.9-12.F.IF.4***

Objective
Solve quadratic equations by graphing.

Vocabulary
quadratic equation

Who uses this?
Dolphin trainers can use solutions of quadratic equations to plan the choreography for their shows. (See Example 2.)

Every quadratic function has a related *quadratic equation*. A **quadratic equation** is an equation that can be written in the standard form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$.

When writing a quadratic function as its related quadratic equation, you replace y with 0.

$$y = ax^2 + bx + c$$
$$0 = ax^2 + bx + c$$
$$ax^2 + bx + c = 0$$

One way to solve a quadratic equation in standard form is to graph the related function and find the x-values where $y = 0$. In other words, find the zeros of the related function. Recall that a quadratic function may have two, one, or no zeros.

Know it! Note

Solving Quadratic Equations by Graphing
Step 1 Write the related function.
Step 2 Graph the related function.
Step 3 Find the zeros of the related function.

EXAMPLE 1 **Solving Quadratic Equations by Graphing**

Solve each equation by graphing the related function.

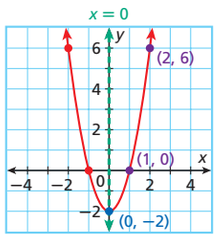
A $2x^2 - 2 = 0$

Step 1 Write the related function.

$2x^2 - 2 = y$, or $y = 2x^2 + 0x - 2$

Step 2 Graph the function.

- The axis of symmetry is $x = 0$.
- The vertex is $(0, -2)$.
- Two other points are $(1, 0)$ and $(2, 6)$.
- Graph the points and reflect them across the axis of symmetry.



Step 3 Find the zeros.

The zeros appear to be -1 and 1.

Check

$2x^2 - 2 = 0$	
$2(-1)^2 - 2$	0
$2(1) - 2$	0
$2 - 2$	0
0	0 ✓

Substitute -1 and 1 for x in the quadratic equation.

$2x^2 - 2 = 0$	
$2(1)^2 - 2$	0
$2(1) - 2$	0
$2 - 2$	0
0	0 ✓

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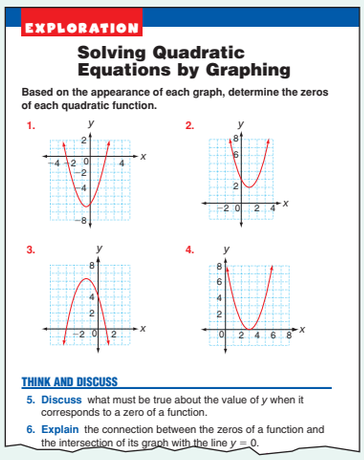
1 Introduce



Motivate

Display the graph of a quadratic function with integer zeros, such as $y = 2x^2 + 14x + 24$. Point out that the zeros of the function are the x-values that correspond to a y-value of 0. Remind students that a quadratic function can have two, one, or no real zeros. Have students find the zeros of the function displayed. -3 and -4 Tell students that in this lesson they will learn to solve quadratic equations by graphing.

Explorations and answers are provided in your online resources.

Solve each equation by graphing the related function.

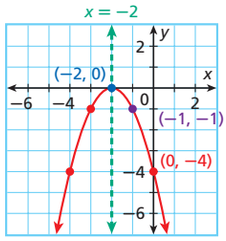
B $-x^2 - 4x - 4 = 0$

Step 1 Write the related function.

$y = -x^2 - 4x - 4$

Step 2 Graph the function.

- The axis of symmetry is $x = -2$.
- The vertex is $(-2, 0)$.
- The y-intercept is -4.
- Another point is $(-1, -1)$.
- Graph the points and **reflect** them across the axis of symmetry.



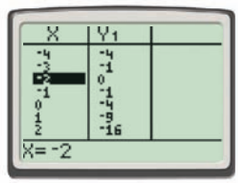
Step 3 Find the zeros.

The only zero appears to be -2.

Check

$$-x^2 - 4x - 4 = 0$$
$$-(-2)^2 - 4(-2) - 4 \mid 0$$
$$-(4) + 8 - 4 \mid 0$$
$$4 - 4 \mid 0$$
$$0 \mid 0 \checkmark$$

You can also confirm the solution by using the **Table** function. Enter the equation and press 2nd TABLE GRAPH. When $y = 0$, $x = -2$. The x-intercept is -2.



C $x^2 + 5 = 4x$

Step 1 Write the related function.

$x^2 - 4x + 5 = 0$

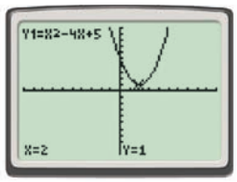
$y = x^2 - 4x + 5$

Step 2 Graph the function.

Use a graphing calculator.

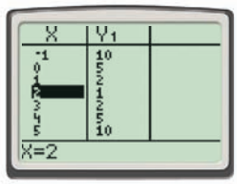
Step 3 Find the zeros.

The function appears to have no zeros.



The equation has no real-number solutions.

Check reasonableness Use the table function.



There are no zeros in the Y1 column. Also, the signs of the values in this column do not change. The function appears to have no zeros.

CHECK IT OUT!

Solve each equation by graphing the related function.

1a. $x^2 - 8x - 16 = 2x^2$ -4

1b. $6x + 10 = -x^2$ no real solutions

1c. $-x^2 + 4 = 0$ $-2, 2$

COMMON ERROR ALERT

Students may try to identify a, b, and c without first writing the equation in the form $ax^2 + bx + c = 0$. Remind students to rewrite the equation in the form $ax^2 + bx + c = 0$ so that the correct signs of each coefficient are identified.

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Additional Examples

Example 1

Solve each equation by graphing the related function.

A. $2x^2 - 18 = 0$ $3, -3$

B. $-12x + 18 = -2x^2$ 3

C. $2x^2 + 4x = -3$ no real solutions

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- If the vertex has a y-coordinate of 0, what does that tell you about the number of zeros of the function?
- Can a quadratic equation have more than two solutions?

Teaching Tip **Critical Thinking** Explain to students how to determine that the equation in **Example 1C** has no real solutions without a graphing calculator. First find the vertex. The vertex, $(2, 1)$, is above the x-axis, and because $a > 0$, the parabola opens upward. This means that the parabola never touches the x-axis and there are no real solutions.

Teaching Tip **Reading Math** Be sure that students understand that a quadratic equation in *standard form* means a quadratic equation in the form $ax^2 + bx + c = 0$. This is the first lesson in which that phrase is used.

2 Teach

Guided Instruction

Emphasize to students that solving an equation by graphing may not always yield exact solutions. Although graphing calculators can do much of the graphing work, the student will need to be familiar with what the function should look like and how many zeros it will have before using a graphing calculator. Tell students that later in this chapter they will learn methods for finding precise solutions and that they may use graphing as a way to support their solutions found by those methods.

Reaching All Learners

Through Multiple Representations

Show students how the equation $x^2 + x = 6$ can also be solved by graphing both sides of the equation as $y = x^2 + x$ and $y = 6$. By looking at where the two graphs intersect, you can find the x-values for which $y = 6$. $2, -3$

Discuss with students how solving $x^2 + x = 6$ by finding the zeros of the related function $f(x) = x^2 + x - 6$ is the same as finding the points where the function intersects with the x-axis, or $y = 0$.

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Additional Examples

Example 2

A frog jumps straight up from the ground. The quadratic function $f(t) = -16t^2 + 12t$ models the frog's height in feet above the ground after t seconds. About how long is the frog in the air? 0.75 s

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 2

- What are the reasonable domain and range values for this function?
- How do you know which zero is the answer to the question?

Teaching Tip **Critical Thinking** Have students solve $x^2 + x - 6 = 0$ by graphing the related function. The solutions are 2 and −3. Ask students to predict what the graph of $-x^2 - x + 6 = 0$ will look like. The graph will be a reflection across the x-axis. Then have students solve $-x^2 - x + 6 = 0$ by graphing the function. Are the solutions the same? yes What is the conclusion to be drawn, and why is it important? The opposite of a function will have the same solutions and will be a reflection across the x-axis.

EXAMPLE 2 ***Aquatics Application***

A dolphin jumps out of the water. The quadratic function $y = -16x^2 + 20x$ models the dolphin's height above the water after x seconds. How long is the dolphin out of the water?

When the dolphin leaves the water, its height is 0, and when the dolphin reenters the water, its height is 0. So solve $0 = -16x^2 + 20x$ to find the times when the dolphin leaves and reenters the water.

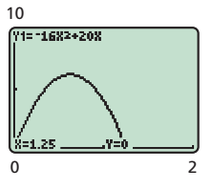
Step 1 Write the related function.

$0 = -16x^2 + 20x$

$y = -16x^2 + 20x$

Step 2 Graph the function.

Use a graphing calculator.



Step 3 Use TRACE to estimate the zeros.

The zeros appear to be 0 and 1.25.

The dolphin leaves the water at 0 seconds and reenters the water at 1.25 seconds.

The dolphin is out of the water for 1.25 seconds.

Check $0 = -16x^2 + 20x$

0	$-16(1.25)^2 + 20(1.25)$
0	$-16(1.5625) + 25$
0	$-25 + 25$
0	0 ✓

Substitute 1.25 for x in the quadratic equation.

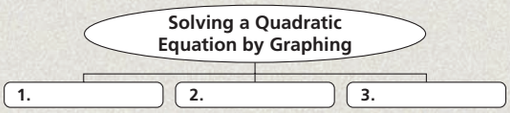
CHECK IT OUT! **2. What if...?** Another dolphin jumps out of the water. The quadratic function $y = -16x^2 + 32x$ models the dolphin's height above the water after x seconds. How long is the dolphin out of the water? 2 s

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Describe the graph of a quadratic function whose related quadratic equation has only one solution.
2. Describe the graph of a quadratic function whose related quadratic equation has no real solutions.
3. Describe the graph of a quadratic function whose related quadratic equation has two solutions.
4. **GET ORGANIZED** Copy and complete the graphic organizer. In each of the boxes, write the steps for solving quadratic equations by graphing.

Know it! Note



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3 Close

Summarize

Review with students the steps for solving quadratic equations by graphing.

1. Write the equation in the form $ax^2 + bx + c = 0$.
2. Write the related function.
3. Graph the function.
4. Find the x-intercepts.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. The vertex is on the x-axis.
2. Either the vertex is above the x-axis and the graph opens upward, or the vertex is below the x-axis and the graph opens downward.
3. Either the vertex is above the x-axis and the graph opens downward, or the vertex is below the x-axis and the graph opens upward.
4. See Additional Answers.

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Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. **Vocabulary** Write two words related to the graph of a quadratic function that can be used to find the solution of the related *quadratic equation*. zeros, *x*-intercepts

SEE EXAMPLE 1

Solve each equation by graphing the related function.

2. $x^2 - 4 = 0$ −2, 2
3. $x^2 = 16$ −4, 4
4. $-2x^2 - 6 = 0$ no real solutions
5. $-x^2 + 12x - 36 = 0$ 6
6. $-x^2 = -9$ −3, 3
7. $2x^2 = 3x^2 - 2x - 8$ −2, 4
8. $x^2 - 6x + 9 = 0$ 3
9. $8x = -4x^2 - 4$ −1
10. $x^2 + 5x + 4 = 0$ −4, −1
11. $x^2 + 2 = 0$ no real solutions
12. $x^2 - 6x = 7$ −1, 7
13. $x^2 + 5x = -8$ no real solutions

SEE EXAMPLE 2

14. **Sports** A baseball coach uses a pitching machine to simulate pop flies during practice. The quadratic function $y = -16x^2 + 80x$ models the height of the baseball after x seconds. How long is the baseball in the air? 5 s

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
15–23	1
24	2

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

Solve each equation by graphing the related function.

15. $-x^2 + 16 = 0$ −4, 4
16. $3x^2 = -7$ no real solutions
17. $5x^2 - 12x + 10 = x^2 + 10x$ $\frac{1}{2}$, 5
18. $x^2 + 10x + 25 = 0$ −5
19. $-4x^2 - 24x = 36$ −3
20. $-9x^2 + 10x - 9 = -8x$ 1
21. $-x^2 - 1 = 0$ no real solutions
22. $3x^2 - 27 = 0$ −3, 3
23. $4x^2 - 4x + 5 = 2x^2$ no real solutions
24. **Geography** Yosemite Falls in California is made of three smaller falls. The upper fall drops 1450 feet. The height h in feet of a water droplet falling from the upper fall to the next fall is modeled by $h(t) = -16t^2 + 1450$, where t is the time in seconds after the initial fall. Estimate the time it takes for the droplet to reach the next cascade. about 9.5 s

Tell whether each statement is always, sometimes, or never true.

25. If the graph of a quadratic function has its vertex at the origin, then the related quadratic equation has exactly one solution. always
26. If the graph of a quadratic function opens upward, then the related quadratic equation has two solutions. sometimes
27. If the graph of a quadratic function has its vertex on the x-axis, then the related quadratic equation has exactly one solution. always
28. If the graph of a quadratic function has its vertex in the first quadrant, then the related quadratic equation has two solutions. sometimes
29. A quadratic equation in the form $ax^2 - c = 0$, where $a < 0$ and $c > 0$, has two solutions. never

30. **Graphing Calculator** A fireworks shell is fired from a mortar. Its height is modeled by the function $h(t) = -16(t - 7)^2 + 784$, where t is the time in seconds and h is the height in feet.
    a. Graph the function.
    b. If the shell is supposed to explode at its maximum height, at what height should it explode? 784 ft
    c. If the shell does not explode, how long will it take to return to the ground? 14 s

8-5 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

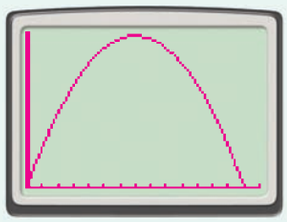
If you finished Examples 1–2

Basic 15–36, 40–43
Average 15–31, 37–43
Advanced 15–31, 37–47

Homework Quick Check
Quickly check key concepts.
Exercises: 18, 20, 22, 24, 30, 31

Answers

30a.



MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 14, 24, 31, 33, 37, 40

Construct viable arguments and critique the reasoning of others. Exercises 38–39

Use appropriate tools strategically. Exercises 30, 32, 34–36, 44–47

Learn It Online
State Resources Online

MULTI-STEP TEST PREP **Exercise 40** involves solving a quadratic equation by graphing to determine the amount of time an object is in the air and then analyzing the graph to find other information. This exercise prepares students for the Multi-Step Test Prep.

Answers

31c. Possible answer: According to the graph, $x = 4$ is a solution to the related equation, but $x = 4$ is not a solution of $0 = -5x^2 + 10x$.

37a. $x - 4$, $x - 8$

b. $0 = x^2 - 24x + 80$

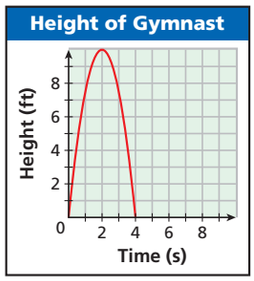
c. 4 and 20

d. Two possible lengths for the hypotenuse; no; if $x = 4$, then the length of one leg would be 0 and the other would be negative, which is impossible.

38. Look for places where the output values are 0. The corresponding input values are the solutions of the related quadratic equation. If there are no output values equal to 0, then look for sign changes. The solution is between the input values that correspond to the y-values with the sign change.

39. The graph of $y = ax^2 - c$, where $a > 0$ and $c > 0$, will always have two solutions because it is a parabola that opens upward whose vertex is below the origin. So it will always cross the x-axis in two places. By contrast, the graph of $y = ax^2 + c$, where $a > 0$ and $c > 0$, opens upward but its vertex is above the origin. Therefore, it will never cross the x-axis.

31. Athletics The graph shows the height y in feet of a gymnast jumping off a vault after x seconds.



- **a.** How long does the gymnast stay in the air? **4 s**
- **b.** What is the maximum height that the gymnast reaches? **10 ft**
- **c.** Explain why the function $y = -5x^2 + 10x$ cannot accurately model the gymnast's motion.

32. Graphing Calculator Use a graphing calculator to solve the equation $x^2 = x + 12$ by graphing $y_1 = x^2$ and $y_2 = x + 12$ and finding the x-coordinates of the points of intersection. (*Hint:* Find the points of intersection by using the 2nd CALC TRACE function after graphing.) **−3, 4**

LINK Biology

Some species of kangaroos are able to jump 30 feet in distance and 6 feet in height.

33. Biology The quadratic function $y = -5x^2 + 7x$ approximates the height y in meters of a kangaroo x seconds after it has jumped. How long does it take the kangaroo to return to the ground? **1.4 s**

For Exercises 34–36, use the table to determine the solutions of the related quadratic equation.

34. **−1**

x	y
−2	−1
−1	0
0	−1
1	−4
2	−9

35. **−1, 1**

x	y
−2	−6
−1	0
0	2
1	0
2	−6

36. **no real solutions**

x	y
−2	6
−1	3
0	2
1	3
2	6

40d. **84 ft; yes, at 1.5 s; 3.5 is 1 unit right of the axis of symmetry ($x = 2.5$). The ball will have the same height at the time represented by the point 1 unit left of the axis of symmetry.**

37. Geometry The hypotenuse of a right triangle is 4 cm longer than one leg and 8 cm longer than the other leg. Let x represent the length of the hypotenuse.

- **a.** Write an expression for the length of each leg in terms of x.
- **b.** Use the Pythagorean Theorem to write an equation that can be solved for x.
- **c.** Find the solutions of your equation from part **b.**
- **d. Critical Thinking** What do the solutions of your equation represent? Are both solutions reasonable? Explain.

38. Write About It Explain how to find solutions of a quadratic equation by analyzing a table of values.

39. Critical Thinking Explain why a quadratic equation in the form $ax^2 - c = 0$, where $a > 0$ and $c > 0$, will always have two solutions. Explain why a quadratic equation in the form $ax^2 + c = 0$, where $a > 0$ and $c > 0$, will never have any real-number solutions.

MULTI-STEP TEST PREP

40. The quadratic equation $0 = -16t^2 + 80t$ gives the time t in seconds when a golf ball is at height 0 feet.

- **a.** How long is the golf ball in the air? **5 s**
- **b.** What is the maximum height of the golf ball? **100 ft**
- **c.** After how many seconds is the ball at its maximum height? **2.5 s**
- **d.** What is the height of the ball after 3.5 seconds? Is there another time when the ball reaches that height? Explain. **See margin above.**

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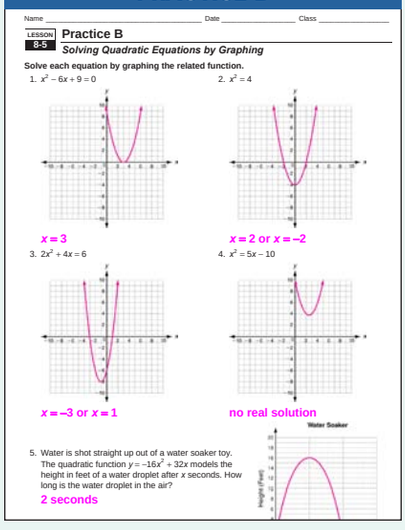
PRACTICE A
PRACTICE C
PRACTICE B



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LESSON 8-5 **Practice B**

Solving Quadratic Equations by Graphing

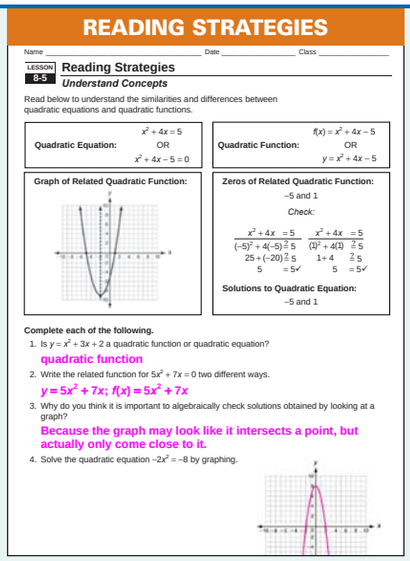
Solve each equation by graphing the related function.

1. $x^2 - 6x + 9 = 0$ — $x = 3$
2. $x^2 = 4$ — $x = 2$ or $x = -2$
3. $2x^2 + 4x = 6$ — $x = -3$ or $x = 1$
4. $x^2 = 5x - 10$ — no real solution
5. Water is shot straight up out of a water soaker toy. The quadratic function $y = -16x^2 + 32x$ models the height in feet of a water droplet after x seconds. How long is the water droplet in the air? **2 seconds**

Water Soaker

Height (Feet)

READING STRATEGIES



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LESSON 8-5 **Reading Strategies**

Understand Concepts

Read below to understand the similarities and differences between quadratic equations and quadratic functions.

Quadratic Equation:	Quadratic Function:
$x^2 + 4x = 5$ OR $x^2 + 4x - 5 = 0$	$f(x) = x^2 + 4x - 5$ OR $y = x^2 + 4x - 5$
Graph of Related Quadratic Function:	Zeros of Related Quadratic Function: −5 and 1

Check:

$\frac{x^2 + 4x = 5}{(-5)^2 + 4(-5) \stackrel{?}{=} 5}$ $\frac{x^2 + 4x = 5}{(1)^2 + 4(1) \stackrel{?}{=} 5}$

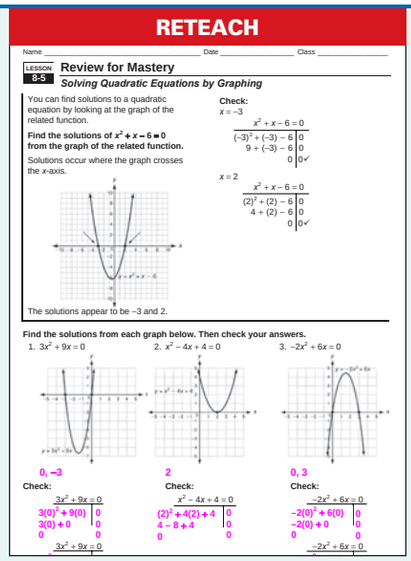
$\frac{25 + (-20) \stackrel{?}{=} 5}{5 = 5\checkmark}$ $\frac{1 + 4 \stackrel{?}{=} 5}{5 = 5\checkmark}$

Solutions to Quadratic Equation: −5 and 1

Complete each of the following.

1. Is $y = x^2 + 3x + 2$ a quadratic function or quadratic equation? **quadratic function**
2. Write the related function for $5x^2 + 7x = 0$ two different ways. **$y = 5x^2 + 7x$; $f(x) = 5x^2 + 7x$**
3. Why do you think it is important to algebraically check solutions obtained by looking at a graph? **Because the graph may look like it intersects a point, but actually only come close to it.**
4. Solve the quadratic equation $-2x^2 = -8$ by graphing.

RETEACH



Name _____ Date _____ Class _____

LESSON 8-5 **Review for Mastery**

Solving Quadratic Equations by Graphing

You can find solutions to a quadratic equation by looking at the graph of the related function.

Find the solutions of $x^2 + x - 6 = 0$ from the graph of the related function.

Solutions occur where the graph crosses the x-axis.

The solutions appear to be −3 and 2.

Check:

$x = -3$: $x^2 + x - 6 = 0$; $(-3)^2 + (-3) - 6 \mid 0$; $9 + (-3) - 6 \mid 0$; $0 \mid 0\checkmark$

$x = 2$: $x^2 + x - 6 = 0$; $(2)^2 + (2) - 6 \mid 0$; $4 + (2) - 6 \mid 0$; $0 \mid 0\checkmark$

Find the solutions from each graph below. Then check your answers.

1. $3x^2 + 9x = 0$ — **0, −3**
2. $x^2 - 4x + 4 = 0$ — **2**
3. $-2x^2 + 6x = 0$ — **0, 3**

Check:

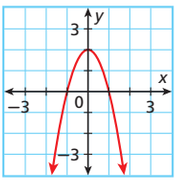
$3x^2 + 9x = 0$; $3(0)^2 + 9(0) \mid 0$; $3(0) + 0 \mid 0$; $0 \mid 0$

$x^2 - 4x + 4 = 0$; $(2)^2 + 4(2) + 4 \mid 0$; $4 - 8 + 4 \mid 0$; $0 \mid 0$

$-2x^2 + 6x = 0$; $-2(0)^2 + 6(0) \mid 0$; $-2(0) + 0 \mid 0$; $0 \mid 0$

TEST PREP

41. Use the graph to find the number of solutions of $-2x^2 + 2 = 0$.

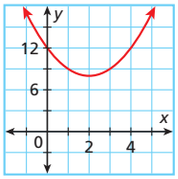


Ⓐ 0 Ⓒ 2

Ⓑ 1 Ⓓ 3

42. Which graph could be used to find the solutions of $x^2 = -4x + 12$?

Ⓕ



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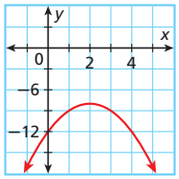

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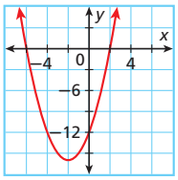

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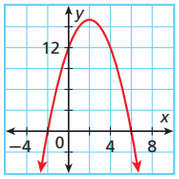
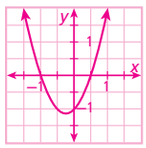


43.



−1 and $\frac{1}{2}$; the *x*-intercepts are the solutions of the equation.

43. Short Response Find the solutions of $2x^2 + x - 1 = 0$ by graphing. Explain how the graph of the related function shows the solutions of the equation.

CHALLENGE AND EXTEND

Graphing Calculator Use a graphing calculator to approximate the solutions of each quadratic equation.

44. $\frac{5}{16}x + \frac{1}{4}x^2 = \frac{3}{5}$ $x \approx -2.3$ or $x \approx 1$

45. $1200x^2 - 650x - 100 = -200x - 175$ no real solutions

46. $\frac{1}{5}x + \frac{3}{4}x^2 = \frac{7}{12}$ $x \approx -1$ or $x \approx 0.75$

47. $400x^2 - 100 = -300x + 456$ $x \approx -1.6$ or $x \approx 0.86$

TEST PREP DOCTOR In **Exercise 42,** remind students to consider the equation in the form $ax^2 + bx + c = 0$. Because $a > 0$, the graph opens upward, so **H** and **J** can be eliminated. By finding that the *y*-intercept of the related function is −12, **F** can also be eliminated.

Journal

Have students explain how to find solutions to a quadratic equation by graphing the related function.

ALTERNATIVE ASSESSMENT

Have students write a quadratic equation with two solutions. They should then solve by graphing. Then have them change either *a*, *b*, or *c* so that the equation has no solutions.

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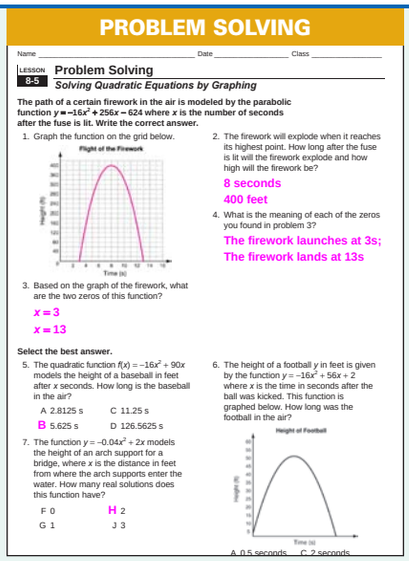
8-5 Lesson Quiz

Solve each equation by graphing the related function.

1. $3x^2 - 12 = 0$ 2, −2
2. $x^2 + 2x = 8$ −4, 2
3. $3x - 5 = x^2$ no real solutions
4. $3x^2 + 3 = 6x$ 1
5. A rocket is shot straight up from the ground. The quadratic function $f(t) = -16t^2 + 96t$ models the rocket's height in feet above the ground after *t* seconds. How long does it take for the rocket to return to the ground? 6 s

Also available online

PROBLEM SOLVING



Name _____ Date _____ Class _____

LESSON 8-5 **Problem Solving**

Solving Quadratic Equations by Graphing

The path of a certain firework in the air is modeled by the parabolic function $y = -16x^2 + 256x - 624$ where x is the number of seconds after the fuse is lit. Write the correct answer.

1. Graph the function on the grid below.

Flight of the Firework

Height (ft) 400 360 320 280 240 200 160 120 80 40

0 2 4 6 8 10 12 14 16 18

Time (s)

2. The firework will explode when it reaches its highest point. How long after the fuse is lit will the firework explode and how high will the firework be?

8 seconds

400 feet

3. Based on the graph of the firework, what are the two zeros of this function?

$x = 3$

$x = 13$

4. What is the meaning of each of the zeros you found in problem 3?

The firework launches at 3s;

The firework lands at 13s

Select the best answer.

5. The quadratic function $f(x) = -16x^2 + 90x$ models the height of a baseball in feet after x seconds. How long is the baseball in the air?

A 2.8125 s C 11.25 s

B 5.625 s D 126.5625 s

6. The height of a football y in feet is given by the function $y = -16x^2 + 56x + 2$ where x is the time in seconds after the ball was kicked. This function is graphed below. How long was the football in the air?

Height of Football

Height (ft) 60 55 50 45 40 35 30 25 20 15 10 5

0 0.5 1 1.5 2 2.5 3 3.5 4

Time (s)

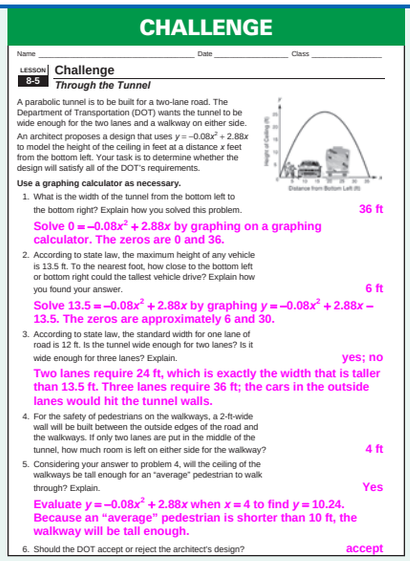
A 0.5 seconds C 2 seconds

7. The function $y = -0.04x^2 + 2x$ models the height of an arch support for a bridge, where x is the distance in feet from where the arch supports enter the water. How many real solutions does this function have?

F 0 H 2

G 1 J 3

CHALLENGE



Name _____ Date _____ Class _____

LESSON 8-5 **Challenge**

Through the Tunnel

A parabolic tunnel is to be built for a two-lane road. The Department of Transportation (DOT) wants the tunnel to be wide enough for the two lanes and a walkway on either side.

An architect proposes a design that uses $y = -0.08x^2 + 2.88x$ to model the height of the ceiling in feet at a distance x feet from the bottom left. Your task is to determine whether the design will satisfy all of the DOT's requirements.

Height of Ceiling (ft) 20 18 16 14 12 10 8 6 4 2

0 2 4 6 8 10 12 14 16 18 20

Distance from Bottom Left (ft)

Use a graphing calculator as necessary.

1. What is the width of the tunnel from the bottom left to the bottom right? Explain how you solved this problem.

36 ft

Solve $0 = -0.08x^2 + 2.88x$ by graphing on a graphing calculator. The zeros are 0 and 36.

2. According to state law, the maximum height of any vehicle is 13.5 ft. To the nearest foot, how close to the bottom left or bottom right could the tallest vehicle drive? Explain how you found your answer.

6 ft

Solve $13.5 = -0.08x^2 + 2.88x$ by graphing $y = -0.08x^2 + 2.88x - 13.5$. The zeros are approximately 6 and 30.

3. According to state law, the standard width for one lane of road is 12 ft. Is the tunnel wide enough for two lanes? Is it wide enough for three lanes? Explain.

yes; no

Two lanes require 24 ft, which is exactly the width that is taller than 13.5 ft. Three lanes require 36 ft; the cars in the outside lanes would hit the tunnel walls.

4. For the safety of pedestrians on the walkways, a 2-ft-wide wall will be built between the outside edges of the road and the walkways. If only two lanes are put in the middle of the tunnel, how much room is left on either side for the walkway?

4 ft

5. Considering your answer to problem 4, will the ceiling of the walkways be tall enough for an "average" pedestrian to walk through? Explain.

Yes

Evaluate $y = -0.08x^2 + 2.88x$ when $x = 4$ to find $y = 10.24$. Because an "average" pedestrian is shorter than 10 ft, the walkway will be tall enough.

6. Should the DOT accept or reject the architect's design?

accept

Organizer

Use with the lesson *Solving Quadratic Equations by Graphing*

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use tables and graphs on a graphing calculator to explore the relationships between roots, zeros, and *x*-intercepts.

Materials: graphing calculator

Online Edition
Graphing Calculator, TechKeys

Countdown

Resources

Technology Lab Activities
Lab Recording Sheet

Teach

Discuss

Discuss how to use sign changes in *y*-values to approximate the zeros from a table when 0 does not appear in the *y*-column.

Close

Key Concept

A quadratic equation can be solved by using a table of values for the related function and locating the zeros of the function, which are the solutions, or roots, of the equation.

Assessment

Journal Have students explain how to use a table to solve a quadratic equation with noninteger solutions.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.A.REI.11 Explain why the *x*- coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values,….*
Also **CC.9-12.A.REI.4**

Learn It Online
State Resources Online

8-5 Technology LAB

Use with Solving Quadratic Equations by Graphing

Explore Roots, Zeros, and *x*-Intercepts

The solutions, or *roots*, of a quadratic equation are the *x*-intercepts, or zeros, of the related quadratic function. You can use tables or graphs on a graphing calculator to understand the connections between zeros, roots, and *x*-intercepts.

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

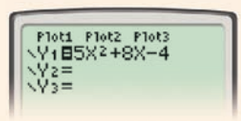
CC.9-12.A.REI.11 Explain why the *x*- coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$;….* *Also* **CC.9-12.A.REI.4**

Learn It Online
Lab Resources Online

Activity 1

Solve $5x^2 + 8x - 4 = 0$ by using a table.

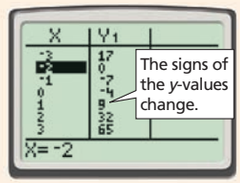
1. Enter the related function in Y_1.



2. Press 2nd TABLE GRAPH to use the **TABLE** function.

3. Scroll through the values by using ▲ and ▼. Look for values of 0 in the Y_1 column. The corresponding *x*-value is a zero of the function. There is one zero at −2.

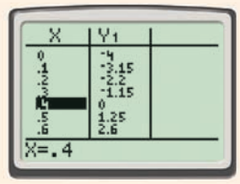
   Also look for places where the signs of nonzero *y*-values change. There is a zero between the corresponding *x*-values. So there is another zero somewhere between 0 and 1.



4. To get a better estimate of the zero, change the table settings. Press 2nd TBLSET WINDOW to view the **TABLE SETUP** screen. Set **TblStart = 0** and the step value **△Tbl = .1**. Press 2nd TABLE GRAPH to see the table again. The table will show you more *x*-values between 0 and 1.

5. Scroll through the values by using ▲ and ▼. The second zero is at 0.4.

   The zeros of the function, −2 and 0.4, are the solutions, or roots, of the equation $5x^2 + 8x - 4 = 0$. Check the solutions algebraically.



Check

$5x^2 + 8x - 4 = 0$	
$5(-2)^2 + 8(-2) - 4$	0
$5(4) - 16 - 4$	0
$20 - 16 - 4$	0
0	0 ✓

$5x^2 + 8x - 4 = 0$	
$5(0.4)^2 + 8(0.4) - 4$	0
$5(0.16) + 3.2 - 4$	0
$0.8 + 3.2 - 4$	0
0	0 ✓

Try This

Solve each equation by using a table.

1. $x^2 - 4x - 5 = 0$
2. $x^2 - x - 6 = 0$
3. $2x^2 + x - 1 = 0$
4. $5x^2 - 6x - 8 = 0$
5. **Critical Thinking** How would you find the zero of a function that showed a sign change in the *y*-values between the *x*-values 1.2 and 1.3?
6. **Make a Conjecture** If you scrolled up and down the list and found only positive *y*-values, what might you conclude?

Teaching Tip

Technology When changing the increments of *x* with the **TBLSET** function in **Activity 1,** consider also changing the *x*-value at which the table starts. This can reduce excess scrolling when locating sign changes in *y*. For instance, if you know there is a zero between 1 and 2, start the table at 1.

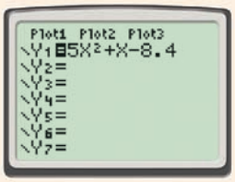
Answers to *Try This*

5. Possible answer: Change **TblStart** to 1.2 and **△Tbl** to 0.01 and scroll until I found the zero.
6. Possible answer: The function has no zeros, and the equation has no real solutions.

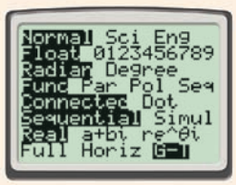
Activity 2

Solve $5x^2 + x - 8.4 = 0$ by using a table and a graph.

1. Enter the related function in Y_1.



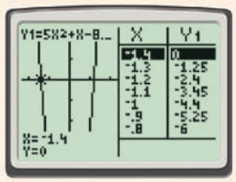
2. To view both the table and the graph at the same time, set your calculator to the Graph-Table mode. Press MODE and select **G-T**.



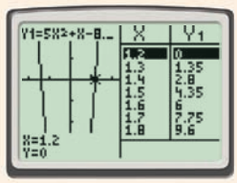
3. Press GRAPH. You should see the graph and the table. Notice that the function appears to have one negative zero and one positive zero near the *y*-axis.

4. To get a closer view of the graph, press ZOOM and select **4:ZDecimal**.

5. Press TRACE. Use ◀ to scroll to find the negative zero. The graph and the table show that the zero is −1.4.



6. Use ▶ to scroll and find the positive zero. The graph and the table show that the zero is 1.2.



The solutions are −1.4 and 1.2. Check the solutions algebraically.

$5x^2 + x - 8.4 = 0$	
$5(-1.4)^2 + (-1.4) - 8.4$	0
$5(1.96) - 1.4 - 8.4$	0
$9.8 - 1.4 - 8.4$	0
0	0 ✓

$5x^2 + x - 8.4 = 0$	
$5(1.2)^2 + (1.2) - 8.4$	0
$5(1.44) + 1.2 - 8.4$	0
$7.2 + 1.2 - 8.4$	0
0	0 ✓

Try This

Solve each equation by using a table and a graph.

7. $2x^2 - x - 3 = 0$ −1, 1.5 **8.** $5x^2 + 13x + 6 = 0$ −2, −0.6 **9.** $10x^2 - 3x - 4 = 0$ −0.5, 0.8 **10.** $x^2 - 2x - 0.96 = 0$ −0.4, 2.4

11. Critical Thinking Suppose that when you graphed a quadratic function, you could see only one side of the graph and one zero. What methods would you use to try to find the other zero? Possible answer: Set the window dimensions wider to see both zeros, and then trace to find the other zero.

8-6 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Solve quadratic equations by factoring.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Find each product.

1. $(x + 2)(x + 7)$ $x^2 + 9x + 14$
2. $(x - 11)(x + 5)$ $x^2 - 6x - 55$
3. $(x - 10)^2$ $x^2 - 20x + 100$

Factor each polynomial.

4. $x^2 + 12x + 35$ $(x + 5)(x + 7)$
5. $x^2 + 2x - 63$ $(x - 7)(x + 9)$
6. $x^2 - 10x + 16$ $(x - 2)(x - 8)$
7. $2x^2 - 16x + 32$ $2(x - 4)^2$

Also available online

Math Humor

Citizen: Why is that quadratic equation lurking about our town?

Sheriff: He's searching for his roots.

CC.9-12.A.REI.4 Solve quadratic equations in one variable.
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation….
CC.9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
a. Factor a quadratic expression to reveal the zeros of the function it defines.

Learn It Online
State Resources Online

COMMON CORE

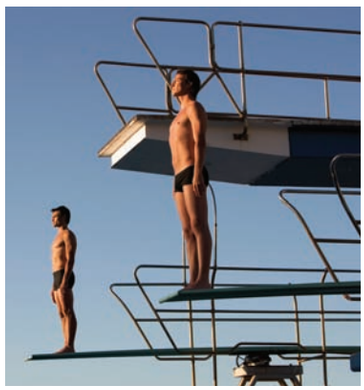
8-6

Solving Quadratic Equations by Factoring

CC.9-12.A.REI.4b Solve quadratic equations by … factoring…. *Also* **CC.9-12.A.SSE.3**

Objective
Solve quadratic equations by factoring.

Who uses this?
In order to determine how many seconds she will be in the air, a high diver can use a quadratic equation. (See Example 3.)



You have solved quadratic equations by graphing. Another method used to solve quadratic equations is to factor and use the Zero Product Property.

Know it! Note

Zero Product Property

For all real numbers a and b,

WORDS	NUMBERS	ALGEBRA
If the product of two quantities equals zero, at least one of the quantities equals zero.	$3(0) = 0$ $0(4) = 0$	If $ab = 0$, then $a = 0$ or $b = 0$.

EXAMPLE 1 **Using the Zero Product Property**

Use the Zero Product Property to solve each equation. Check your answer.

A $(x - 3)(x + 7) = 0$

$x - 3 = 0$ or $x + 7 = 0$ *Use the Zero Product Property.*
$x = 3$ or $x = -7$ *Solve each equation.*

The solutions are 3 and −7.

Check

$(x - 3)(x + 7) = 0$	
$(3 - 3)(3 + 7)$	0
$(0)(10)$	0
0	0 ✓

Substitute each solution for x into the original equation.

$(x - 3)(x + 7) = 0$	
$(-7 - 3)(-7 + 7)$	0
$(-10)(0)$	0
0	0 ✓

B $(x)(x - 5) = 0$

$x = 0$ or $x - 5 = 0$ *Use the Zero Product Property.*
$x = 5$ *Solve the second equation.*

The solutions are 0 and 5.

Check

$(x)(x - 5) = 0$	
$(0)(0 - 5)$	0
$(0)(-5)$	0
0	0 ✓

Substitute each solution for x into the original equation.

$(x)(x - 5) = 0$	
$(5)(5 - 5)$	0
$(5)(0)$	0
0	0 ✓

CHECK IT OUT!

Use the Zero Product Property to solve each equation. Check your answer.

1a. $(x)(x + 4) = 0$ **0, −4**

1b. $(x + 4)(x - 3) = 0$ **−4, 3**

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1 Introduce

EXPLORATION

Solving Quadratic Equations by Factoring

You will need a graphing calculator for this Exploration.

1. Graph each function listed in the table, and use the graph to find the zeros of the function. Record the zeros and the number of zeros.

Function	Zeros	Number of Zeros
$y = (x - 2)(x + 1)$	2, −1	2
$y = (x + 3)(x + 7)$	−3, −7	2
$y = (x + 6)(x - 1)$	−6, 1	2
$y = (x - 2.5)(x - 5)$	2.5, 5	2
$y = (x - 4)^2$	4	1
$y = (x + 3)^2$	−3	1

2. Predict the zeros of the function $y = (x - 7)(x + 10)$ without graphing. 7, −10
3. Predict the zeros of the function $y = (x + 8)^2$ without graphing. −8

THINK AND DISCUSS

4. **Explain** how you predicted the zeros of the functions in Problems 2 and 3. The zeros of the functions are the values of a in each factor $(x - a)$.
5. **Describe** any patterns you notice in your table. The zeros of the

Motivate

Display the linear equation $4x = x + 12$ and the quadratic equation $4x = x^2 + 12$. Ask students why they cannot solve for x in the quadratic equation as they would in the linear equation. Possible answer: The variable terms are not like terms and cannot be combined to isolate x.
Tell them that another method is needed to solve quadratic equations algebraically.

Explorations and answers are provided in your online resources.

If a quadratic equation is written in standard form, $ax^2 + bx + c = 0$, you may need to factor before using the Zero Product Property to solve the equation.

EXAMPLE 2 Solving Quadratic Equations by Factoring

Solve each quadratic equation by factoring. Check your answer.

Helpful Hint

You may want to review factoring techniques before solving quadratic equations.

A $x^2 + 7x + 10 = 0$

$(x + 5)(x + 2) = 0$ — *Factor the trinomial.*

$x + 5 = 0$ or $x + 2 = 0$ — *Use the Zero Product Property.*

$x = -5$ or $x = -2$ — *Solve each equation.*

The solutions are -5 and -2.

Check

$x^2 + 7x + 10 = 0$	
$(-5)^2 + 7(-5) + 10$	0
$25 - 35 + 10$	0
0	0 ✓

$x^2 + 7x + 10 = 0$	
$(-2)^2 + 7(-2) + 10$	0
$4 - 14 + 10$	0
0	0 ✓

B $x^2 + 2x = 8$

$x^2 + 2x = 8$

$-8 \quad -8$ — *The equation must be written in standard form. So subtract 8 from both sides.*

$x^2 + 2x - 8 = 0$

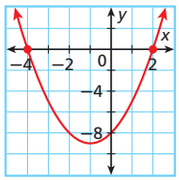
$(x + 4)(x - 2) = 0$ — *Factor the trinomial.*

$x + 4 = 0$ or $x - 2 = 0$ — *Use the Zero Product Property.*

$x = -4$ or $x = 2$ — *Solve each equation.*

The solutions are -4 and 2.

Check Graph the related quadratic function. The zeros of the related function should be the same as the solutions from factoring.



The graph of $y = x^2 + 2x - 8$ shows two zeros that appear to be -4 and 2, the same as the solutions from factoring. ✓

C $x^2 + 2x + 1 = 0$

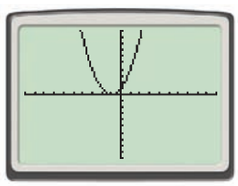
$(x + 1)(x + 1) = 0$ — *Factor the trinomial.*

$x + 1 = 0$ or $x + 1 = 0$ — *Use the Zero Product property.*

$x = -1$ or $x = -1$ — *Solve each equation.*

Both factors result in the same solution, so there is one solution, -1.

Check Use a graphing calculator to graph the related function.



The graph of $y = x^2 + 2x + 1$ shows one zero that appears to be -1, the same as the solution from factoring. ✓

COMMON ERROR ALERT

When solving quadratic equations by factoring, students sometimes choose the constants in the binomial factors as the solutions. For example, students might say that the solutions of $(x + 3)(x + 5) = 0$ are 3 and 5. Remind students to find the value of x that would make each factor equal 0.

$x + 3 = 0$ or $x + 5 = 0$

$x = -3$ $\quad$ $x = -5$

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Additional Examples

Example 1

Use the Zero Product Property to solve each equation. Check your answer.

A. $(x - 7)(x + 2) = 0$ 7, −2

B. $(x - 2)(x) = 0$ 0, 2

Example 2

Solve each quadratic equation by factoring. Check your answer.

A. $x^2 - 6x + 8 = 0$ 4, 2

B. $x^2 + 4x = 21$ −7, 3

C. $x^2 - 12x + 36 = 0$ 6

D. $-2x^2 = 20x + 50$ −5

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- Why do you set each factor equal to zero?
- How can you tell whether an expression is quadratic when it is written in factored form?

EXAMPLE 2

- Why must one side of the equation be equal to zero?
- What types of quadratic equations have only one real solution?
- When the graph of a function crosses the x-axis, what is the value of y?

2 Teach

Guided Instruction

Review multiplying binomials and factoring trinomials, including how to recognize perfect-square trinomials. Remind students that the first step in factoring is always to look for a GCF. Emphasize the importance of not using the Zero Product Property until one side of the equation is zero. Encourage students to substitute solutions into the original equations to check their answers.

Reaching All Learners *Through Critical Thinking*

Show students how to create quadratic equations with given solutions by working backward. For example, if the solutions are -2 and 5, then:

$x = -2$ or $x = 5$

$(x + 2)(x - 5) = 0$

$x^2 - 3x - 10 = 0$

Explain that there are infinitely many equivalent equations that can be generated by performing the same operation on both sides.

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Additional Examples

Example 3

The height in feet above the water of a diver can be modeled by $h(t) = -16t^2 + 8t + 8$, where t is time in seconds after the diver jumps off a platform. Find the time it takes for the diver to reach the water. 1 s

Also available online

Helpful Hint

$(x - 3)(x - 3)$ is a perfect square. Since both factors are the same, you solve only one of them.

Solve each quadratic equation by factoring. Check your answer.

D $-2x^2 = 18 - 12x$

$-2x^2 + 12x - 18 = 0$ *Write the equation in standard form.*

$-2(x^2 - 6x + 9) = 0$ *Factor out the GCF, −2.*

$-2(x - 3)(x - 3) = 0$ *Factor the trinomial.*

$-2 \neq 0$ or $x - 3 = 0$ *Use the Zero Product Property. −2 cannot equal 0.*

$x = 3$ *Solve the remaining equation.*

The only solution is 3.

Check

$-2x^2 = 18 - 12x$	
$-2(3)^2$	$18 - 12(3)$
-18	$18 - 36$
-18	-18 ✓

Substitute 3 into the original equation.

CHECK IT OUT! **Solve each quadratic equation by factoring. Check your answer.**

2a. $x^2 - 6x + 9 = 0$ **3**

2b. $x^2 + 4x = 5$ **1, −5**

2c. $30x = -9x^2 - 25$ $-\frac{5}{3}$

2d. $3x^2 - 4x + 1 = 0$ $\frac{1}{3}$, **1**

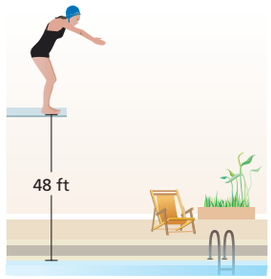
Mathematical Practices — Questioning Strategies

EXAMPLE 3

- What equation do you need to solve?
- What is a reasonable domain and range for this situation?

EXAMPLE 3 *Sports Application*

The height of a diver above the water during a dive can be modeled by $h = -16t^2 + 8t + 48$, where h is height in feet and t is time in seconds. Find the time it takes for the diver to reach the water.



$h = -16t^2 + 8t + 48$

$0 = -16t^2 + 8t + 48$ *The diver reaches the water when h = 0.*

$0 = -8(2t^2 - t - 6)$ *Factor out the GCF, −8.*

$0 = -8(2t + 3)(t - 2)$ *Factor the trinomial.*

$-8 \neq 0, 2t + 3 = 0$ or $t - 2 = 0$ *Use the Zero Product Property.*

$2t = -3$ or $t = 2$ *Solve each equation.*

$t = -\frac{3}{2}$ ✗ *Since time cannot be negative, $-\frac{3}{2}$ does not make sense in this situation.*

It takes the diver 2 seconds to reach the water.

Check

$0 = -16t^2 + 8t + 48$	
0	$-16(2)^2 + 8(2) + 48$
0	$-64 + 16 + 48$
0	0 ✓

Substitute 2 into the original equation.

CHECK IT OUT! **3. What if...?** The equation for the height above the water for another diver can be modeled by $h = -16t^2 + 8t + 24$. Find the time it takes this diver to reach the water. **1.5 s**

3 Close

Summarize

Have students state the first step in solving each equation.

- $(x + 1)(x - 2) = 0$
  Set each factor equal to zero.
- $x^2 + 8x + 7 = 0$
  Factor the left side.
- $2x^2 - 16x + 30 = 0$
  Factor out 2 from each term in the trinomial.
- $x^2 - 9x - 10 = -18$
  Add 18 to each side.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

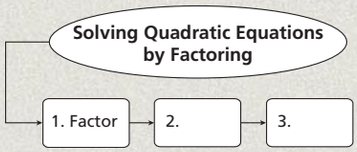
MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Explain two ways to solve $x^2 + x - 6 = 0$. How are these two methods similar?
2. Describe the relationships among the solutions of $x^2 - 4x - 12 = 0$, the zeros and x-intercepts of $y = x^2 - 4x - 12$, and the factors of $x^2 - 4x - 12$.

Know it! Note

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write a step used to solve a quadratic equation by factoring.



8-6 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

SEE EXAMPLE 1

Use the Zero Product Property to solve each equation. Check your answer.

1. $(x + 2)(x - 8) = 0$ $-2, 8$
2. $(x - 6)(x - 5) = 0$ $6, 5$
3. $(x + 7)(x + 9) = 0$ $-7, -9$
4. $(x)(x - 1) = 0$ $0, 1$
5. $(x)(x + 11) = 0$ $-11, 0$
6. $(3x + 2)(4x - 1) = 0$ $-\frac{2}{3}, \frac{1}{4}$

SEE EXAMPLE 2

Solve each quadratic equation by factoring. Check your answer.

7. $x^2 + 4x - 12 = 0$ $-6, 2$
8. $x^2 - 8x - 9 = 0$ $-1, 9$
9. $x^2 - 5x + 6 = 0$ $2, 3$
10. $x^2 - 3x = 10$ $-2, 5$
11. $x^2 + 10x = -16$ $-8, -2$
12. $x^2 + 2x = 15$ $-5, 3$
13. $x^2 - 8x + 16 = 0$ 4
14. $-3x^2 = 18x + 27$ -3
15. $x^2 + 36 = 12x$ 6
16. $2x^2 + 5x - 3 = 0$ $-3, \frac{1}{2}$
17. $2x^2 + 7x + 6 = 0$ $-2, -\frac{3}{2}$
18. $2x^2 + 6x = -18$ no real solutions

SEE EXAMPLE 3

19. **Games** A group of friends tries to keep a beanbag from touching the ground. On one kick, the beanbag's height can be modeled by $h = -16t^2 + 14t + 2$, where h is the height in feet above the ground and t is the time in seconds. Find the time it takes the beanbag to reach the ground. 1 s

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
20–25	1
26–31	2
32	3

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Use the Zero Product Property to solve each equation. Check your answer.

20. $(x - 8)(x + 6) = 0$ $-6, 8$
21. $(x + 4)(x + 7) = 0$ $-4, -7$
22. $(x - 2)(x - 5) = 0$ $2, 5$
23. $(x - 9)(x) = 0$ $9, 0$
24. $(x)(x + 25) = 0$ $0, -25$
25. $(2x + 1)(3x - 1) = 0$ $-\frac{1}{2}, \frac{1}{3}$

Solve each quadratic equation by factoring. Check your answer.

26. $x^2 + 8x + 15 = 0$ $-3, -5$
27. $x^2 - 2x - 8 = 0$ $-2, 4$
28. $x^2 - 4x + 3 = 0$ $1, 3$
29. $3x^2 - 2x - 1 = 0$ $-\frac{1}{3}, 1$
30. $4x^2 - 9x + 2 = 0$ $\frac{1}{4}, 2$
31. $-x^2 = 4x + 4$ -2

Answers to *Think and Discuss*

1. Possible answer: One method would be to factor and set each factor equal to 0. Another method would be to graph and find the x-intercepts of the graph. In both methods, 0 is important. In the factoring method, one side of the equation must be 0. In the graphing method, you identify values of x where $f(x) = 0$.
2. Possible answer: The solutions of the equation are $x = -2$ and $x = 6$. These solutions indicate both the zeros and the x-intercepts of the related function (-2 and 6). The factors of $x^2 - 4x - 12$ are $x - 6$ and $x + 2$. When each of these factors is set equal to 0 and solved for x, the solutions are the same as the solutions of $x^2 - 4x - 12 = 0$.
3. See Additional Answers.

8-6 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–2
Basic 20–31, 33–39, 46
Average 20–31, 33–39, 45–46
Advanced 20–31, 33–38, 45

If you finished Examples 1–3
Basic 20–39, 41–43, 46–49
Average 20–39, 40–44, 46–50
Advanced 20–38, 40–45, 47–61

Homework Quick Check
Quickly check key concepts.
Exercises: 20, 24, 26, 30, 32, 44

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 19, 32, 40–44, 47, 59–61

Construct viable arguments and critique the reasoning of others. Exercises 39, 45–46

Learn It Online
State Resources Online

MULTI-STEP TEST PREP **Exercise 47** involves solving a quadratic equation by factoring to determine the amount of time an object is in the air. This exercise prepares students for the Multi-Step Test Prep.

Answers

40. $x(x + 2) = 24$; 4 and 6 or -6 and -4

45. No; by the Zero Product Property, you can solve $(x - 2)(x + 3) = 0$ by solving $x - 2 = 0$ and $x + 3 = 0$. There is no "Five" Product Property, however, so you cannot solve $(x - 2)(x + 3) = 5$ by solving $x - 2 = 5$ and $x + 3 = 5$.

46. The Zero Product Property states that if the product of two numbers is 0, then at least one of the numbers must be 0. When you factor a quadratic and the product of the factors is 0, at least one of the factors must be 0.

32. Multi-Step The height of a flare can be approximated by the function $h = -16t^2 + 95t + 6$, where h is the height in feet and t is the time in seconds. Find the time it takes the flare to hit the ground. **6 s**

Determine the number of solutions of each equation.

33. $(x + 8)(x + 8) = 0$ **1** **34.** $(x - 3)(x + 3) = 0$ **2** **35.** $(x + 7)^2 = 0$ **1**

36. $3x^2 + 12x + 9 = 0$ **2** **37.** $x^2 + 12x + 40 = 4$ **1** **38.** $(x - 2)^2 = 9$ **2**

39. ERROR ANALYSIS Which solution is incorrect? Explain the error.

A

$x^2 + x - 2 = 0$
$(x - 1)(x + 2) = 0$
$x = 1$ or $x = -2$

B

$x^2 + x - 2 = 0$
$(x - 1)(x + 2) = 0$
$x = -1$ or $x = 2$

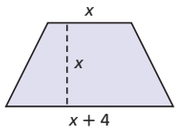
39. B; after factoring the polynomial, you should solve the equations $x - 1 = 0$ and $x + 2 = 0$. The correct solutions are 1 and -2.

40. Number Theory Write an equation that could be used to find two consecutive even integers whose product is 24. Let x represent the first integer. Solve the equation and give the two integers.

41. Geometry The photo shows a traditional thatched house as found in Santana, Madeira, in Portugal. The front of the house is in the shape of a triangle. Suppose the base of the triangle is 1 m less than its height and the area of the triangle is 15 m^2. Find the height of the triangle. (*Hint:* Use $A = \frac{1}{2}bh$.) **6 m**

42. Multi-Step The length of a rectangle is 1 ft less than 3 times the width. The area is 310 ft^2. Find the dimensions of the rectangle. **$10\frac{1}{3}$ ft by 30 ft**

43. Physics The height of a fireworks rocket in meters can be approximated by $h = -5t^2 + 30t$, where h is the height in meters and t is time in seconds. Find the time it takes the rocket to reach the ground after it has been launched. **6 s**



44. Geometry One base of a trapezoid is the same length as the height of the trapezoid. The other base is 4 cm more than the height. The area of the trapezoid is 48 cm^2. Find the length of the shorter base. $\left(\text{Hint: Use } A = \frac{1}{2}h(b_1 + b_2)\right)$. **6 cm**

45. Critical Thinking Can you solve $(x - 2)(x + 3) = 5$ by solving $x - 2 = 5$ and $x + 3 = 5$? Why or why not?

46. Write About It Explain why you set each factor equal to zero when solving a quadratic equation by factoring.

MULTI-STEP TEST PREP

47. A tee box is 48 feet above its fairway. When a golf ball is hit from the tee box with an initial vertical velocity of 32 ft/s, the quadratic equation $0 = -16t^2 + 32t + 48$ gives the time t in seconds when a golf ball is at height 0 feet on the fairway.

a. Solve the quadratic equation by factoring to see how long the ball is in the air. **3 s**

b. What is the height of the ball at 1 second? **64 ft**

c. Is the ball at its maximum height at 1 second? Explain. **Yes; the vertex is (1, 64), which represents the ball's maximum height of 64 ft after 1 s in the air.**

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PRACTICE A

PRACTICE C

PRACTICE B

Name _____ Date _____ Class _____

LESSON 8-6 **Practice B**

Solving Quadratic Equations by Factoring

Use the Zero Product Property to solve each equation. Check your answers.

1. $(x - 1)(x - 5) = 0$
   $x - 1 = 0$ or $x - 5 = 0$
   $x = 1$ or $x = 5$
2. $(x - 2)(x - 9) = 0$
   $x - 2 = 0$ or $x - 9 = 0$
   $x = 2$ or $x = 9$
3. $(x - 2)(x + 4) = 0$
   2; –4
4. $(2x + 1)(x - 6) = 0$
   $-\frac{1}{2}$; 6

Solve each quadratic equation by factoring.

5. $x^2 - 3x = 0$ — 0; 3
6. $x^2 + 4x + 3 = 0$ — –3; –1
7. $x^2 + 5x - 6 = 0$ — 1; –6
8. $x^2 + 11x + 24 = 0$ — –3; –8
9. $x^2 - 12x + 11 = 0$ — 1; 11
10. $x^2 + 18x + 65 = 0$ — –13; –5
11. $x^2 - 4x - 12 = 0$ — 6; –2
12. $x^2 + 11x + 10 = 0$ — –1; –10
13. $x^2 + 12x + 35 = 0$ — –5; –7
14. $2x^2 - 3x - 5 = 0$ — $\frac{5}{2}$; –1
15. $3x^2 - 5x - 2 = 0$ — $-\frac{1}{3}$; 2
16. $x^2 = 3x + 40$ — 8; –5
17. $x^2 - 14 = 5x$ — 7; –2
18. $2x - 1 = -8x^2$ — $\frac{1}{4}$; $-\frac{1}{2}$
19. $x = 10x^2 - 2$ — $-\frac{2}{5}$; $\frac{1}{2}$
20. $2x^2 = 13x + 7$ — 7; $-\frac{1}{2}$
21. $6x^2 + x = 5$ — –1; $\frac{5}{6}$
22. $x^2 = 5x$ — 0; 5
23. The height of a flare fired from the deck of a ship in distress can be modeled by $h = -16t^2 + 104t + 56$, where h is the height of the flare above water and t is the time in seconds. Find the time it takes the flare to hit the water. — 7 seconds

READING STRATEGIES

Name _____ Date _____ Class _____

LESSON 8-6 **Reading Strategies**

Use a Concept Map

Use this concept map to understand the Zero Product Property.

Word Definition

If you multiply two or more quantities, and the answer is 0, at least one of the quantities must be 0.

Algebraic Definition

For all real numbers a and b, if $a \cdot b = 0$, then $a = 0$ or $b = 0$.

Zero Product Property

Examples

If $-4m = 0$, then $m = 0$.

If $x(x + 2) = 0$, then $x = 0$, or $x + 2 = 0$.

If $(t - 5)(t + 1) = 0$, then $t - 5 = 0$, or $t + 1 = 0$.

Application

Use the Zero Product Property to solve quadratic equations.

$x^2 - 5x = 6$
$x^2 - 5x - 6 = 0$
$(x - 6)(x + 1) = 0$
$x - 6 = 0$ or $x + 1 = 0$
$x = 6, x = -1$

Answer each of the following.

1. Look at the algebraic definition in the concept map. Do you think a and b can both be zero at the same time? Explain.
   yes; because $0 \times 0 = 0$
2. Look at the application in the concept map. Why must the expression be factored before using the Zero Product Property?
   Because you need two or more quantities being multiplied
3. Look at the second example in the concept map. If $x + 2 = 0$, what must be the value of x? Why?
   –2, because $-2 + 2 = 0$.
4. Give an algebraic example of the Zero Product Property with three factors.
   If $abc = 0$, then $a = 0$, or $b = 0$, or $c = 0$.

Solve each quadratic equation by using the Zero Product Property.

RETEACH

Name _____ Date _____ Class _____

LESSON 8-6 **Review for Mastery**

Solving Quadratic Equations by Factoring

Quadratic Equations can be solved by factoring and using the Zero Product Property.

If the product of two quantities equals zero, at least one of the quantities must equal zero.

If $(x)(y) = 0$, then $x = 0$ or $y = 0$.

If $(x + 3)(x - 2) = 0$, then $x + 3 = 0$ or $x - 2 = 0$.

Use the Zero Product Property to solve $(x + 8)(x - 5) = 0$. Check your answer.

$(x + 8)(x - 5) = 0$
$x + 8 = 0$ or $x - 5 = 0$
$-8 \quad -8 \quad +5 \quad +5$
$x = -8$ $\quad x = 5$

Check

$x = -8$

$(x + 8)(x - 5) = 0$	
$(-8 + 8)(-8 - 5) = 0$	
$(0)(-13) = 0$	
$0 = 0$	✓

$x = 5$

$(x + 8)(x - 5) = 0$	
$(5 + 8)(5 - 5) = 0$	
$(13)(0) = 0$	
$0 = 0$	✓

Use the Zero Product Property to solve each equation by filling in the boxes below. Then find the solutions. Check your answer.

1. $(x - 6)(x - 3) = 0$
   $x - 6 = 0$ or $x - 3 = 0$
   6; 3
2. $(x + 8)(x - 5) = 0$
   $x + 8 = 0$ or $x - 5 = 0$
   –8; 5
3. $3x(x - 7) = 0$
   $3x = 0$ or $x - 7 = 0$
   0; 7
4. $(2x - 3)(x + 9) = 0$
   $2x - 3 = 0$ or $x + 9 = 0$
   $\frac{3}{2}$; –9
5. $(5x - 1)(x + 2) = 0$
6. $(x + 4)(2 - x) = 0$

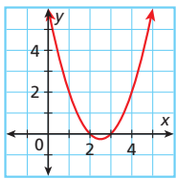
TEST PREP

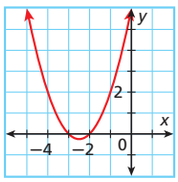
48. What are the solutions to $(x - 1)(2x + 5) = 0$?

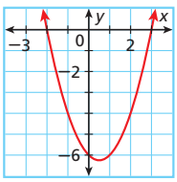
Ⓐ -1 and $\frac{5}{2}$ Ⓒ 1 and $-\frac{5}{2}$

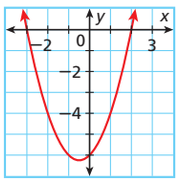
Ⓑ -1 and $\frac{2}{5}$ Ⓓ 1 and $-\frac{2}{5}$

49. Which graph could be used to solve the equation $(x - 3)(x - 2) = 0$?

Ⓕ 

Ⓗ 

Ⓖ 

Ⓙ 

CHALLENGE AND EXTEND

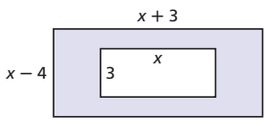
Solve each equation by factoring.

50. $6x^2 + 11x = 10$ $-\frac{5}{2}, \frac{2}{3}$ **51.** $0.2x^2 + 1 = -1.2x$ $-5, -1$ **52.** $\frac{1}{3}x^2 = 2x - 3$ 3

53. $75x - 45 = -30x^2$ $\frac{1}{2}, -3$ **54.** $x^2 = -4(2x + 3)$ $-6, -2$ **55.** $\frac{x(x-3)}{2} = 5$ $-2, 5$

56. $x(x - 10)(x - 3) = 0$ 0, 3, 10 **57.** $x^3 + 2x^2 + x = 0$ $-1, 0$ **58.** $x^3 + 4x^2 = 0$ $-4, 0$

Geometry **Use the diagram for Exercises 59–61.**

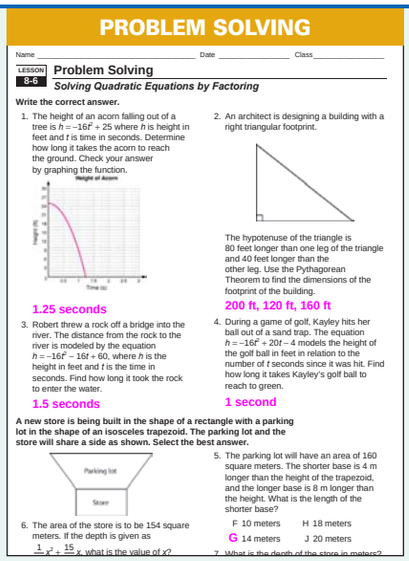


59. Write a polynomial to represent the area of the larger rectangle. $x^2 - x - 12$

60. Write a polynomial to represent the area of the smaller rectangle. $3x$

61. Write a polynomial to represent the area of the shaded region. Then solve for x given that the area of the shaded region is 48 square units. $x^2 - 4x - 12 = 48$; $x = 10$

PROBLEM SOLVING



Name _____ Date _____ Class _____

LESSON 8-6 **Problem Solving**

Solving Quadratic Equations by Factoring

Write the correct answer.

1. The height of an acorn falling out of a tree is $h = -16t^2 + 25$ where h is height in feet and t is time in seconds. Determine how long it takes the acorn to reach the ground. Check your answer by graphing the function.

Height of Acorn

1.25 seconds

2. An architect is designing a building with a right triangular footprint.

The hypotenuse of the triangle is 80 feet longer than one leg of the triangle and 40 feet longer than the other leg. Use the Pythagorean Theorem to find the dimensions of the footprint of the building.

200 ft, 120 ft, 160 ft

3. Robert threw a rock off a bridge into the river. The distance from the rock to the river is modeled by the equation $h = -16t^2 - 16t + 60$, where h is the height in feet and t is the time in seconds. Find how long it took the rock to enter the water.

1.5 seconds

4. During a game of golf, Kayley hits her ball out of a sand trap. The equation $h = -16t^2 + 20t - 4$ models the height of the golf ball in feet in relation to the number of t seconds since it was hit. Find how long it takes Kayley's golf ball to reach to green.

1 second

A new store is being built in the shape of a rectangle with a parking lot in the shape of an isosceles trapezoid. The parking lot and the store will share a side as shown. Select the best answer.

Parking lot

Store

5. The parking lot will have an area of 160 square meters. The shorter base is 4 m longer than the height of the trapezoid, and the longer base is 8 m longer than the height. What is the length of the shorter base?

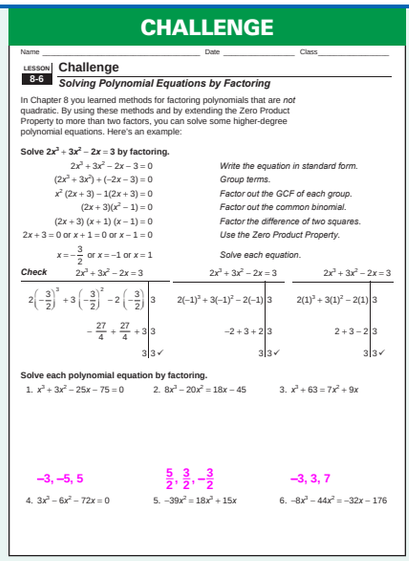
F 10 meters H 18 meters

G 14 meters J 20 meters

6. The area of the store is to be 154 square meters. If the depth is given as $\frac{1}{2}x^2 + \frac{15}{2}x$, what is the value of x?

7. What is the depth of the store in meters?

CHALLENGE



Name _____ Date _____ Class _____

LESSON 8-6 **Challenge**

Solving Polynomial Equations by Factoring

In Chapter 8 you learned methods for factoring polynomials that are *not* quadratic. By using these methods and by extending the Zero Product Property to more than two factors, you can solve some higher-degree polynomial equations. Here's an example:

Solve $2x^3 + 3x^2 - 2x = 3$ by factoring.

$2x^3 + 3x^2 - 2x - 3 = 0$ *Write the equation in standard form.*

$(2x^3 + 3x^2) + (-2x - 3) = 0$ *Group terms.*

$x^2(2x + 3) - 1(2x + 3) = 0$ *Factor out the GCF of each group.*

$(2x + 3)(x^2 - 1) = 0$ *Factor out the common binomial.*

$(2x + 3)(x + 1)(x - 1) = 0$ *Factor the difference of two squares.*

$2x + 3 = 0$ or $x + 1 = 0$ or $x - 1 = 0$ *Use the Zero Product Property.*

$x = -\frac{3}{2}$ or $x = -1$ or $x = 1$ *Solve each equation.*

Check $2x^3 + 3x^2 - 2x = 3$ | $2x^3 + 3x^2 - 2x = 3$ | $2x^3 + 3x^2 - 2x = 3$

$2\left(-\frac{3}{2}\right)^3 + 3\left(-\frac{3}{2}\right)^2 - 2\left(-\frac{3}{2}\right) \mid 3$ | $2(-1)^3 + 3(-1)^2 - 2(-1) \mid 3$ | $2(1)^3 + 3(1)^2 - 2(1) \mid 3$

$-\frac{27}{4} + \frac{27}{4} + 3 \mid 3$ | $-2 + 3 + 2 \mid 3$ | $2 + 3 - 2 \mid 3$

$3 \mid 3$ ✓ | $3 \mid 3$ ✓ | $3 \mid 3$ ✓

Solve each polynomial equation by factoring.

1. $x^3 + 3x^2 - 25x - 75 = 0$ 2. $8x^3 - 20x^2 = 18x - 45$ 3. $x^2 + 63 = 7x^2 + 9x$

$-3, -5, 5$ $\frac{5}{2}, \frac{3}{2}, -\frac{3}{2}$ $-3, 3, 7$

4. $3x^3 - 6x^2 - 72x = 0$ 5. $-39x^2 = 18x^3 + 15x$ 6. $-8x^3 - 44x^2 = -32x - 176$

TEST PREP DOCTOR In **Exercise 49,** students can examine the equation to see that the zeros of the related function must be 3 and 2. Only **F** shows a function with these zeros.

Journal

Explain why you must have 0 on one side of the equation in order to apply the Zero Product Property to solve an equation.

ALTERNATIVE ASSESSMENT

Have students use binomial factors to work backward to create two equivalent quadratic equations with given solutions.

Power Presentations with PowerPoint®

8-6 Lesson Quiz

Use the Zero Product Property to solve each equation. Check your answer.

1. $(x - 10)(x + 5) = 0$ 10, −5

2. $(x + 5)(x) = 0$ −5, 0

Solve each quadratic equation by factoring. Check your answer.

3. $x^2 + 16x + 48 = 0$ −4, −12

4. $x^2 - 11x = -24$ 3, 8

5. $2x^2 + 12x - 14 = 0$ 1, −7

6. $x^2 + 18x + 81 = 0$ −9

7. $-4x^2 = 16x + 16$ −2

8. The height of a rocket launched upward from a 160-foot cliff is modeled by the function $h(t) = -16t^2 + 48t + 160$, where h is height in feet and t is time in seconds. Find the time it takes the rocket to reach the ground at the bottom of the cliff. 5 s

Also available online

8-7 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Solve quadratic equations by using square roots.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Find each square root.

1. $\sqrt{36}$ 6
2. $\sqrt{121}$ 11
3. $-\sqrt{625}$ −25
4. $\sqrt{\frac{4}{25}}$ $\frac{2}{5}$

Solve each equation.

5. $-6x = -60$ 10
6. $\frac{1}{5}x = 16$ 80
7. $2x - 40 = 0$ 20
8. $5x = 3$ $\frac{3}{5}$

Also available online

Math Humor

Mother: Why did you rip up my garden?

Student: My math assignment is to find roots.

COMMON CORE

CC.9-12.A.REI.4 Solve quadratic equations in one variable.
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation….
CC.9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.*
Also **CC.9-12.A.CED.1*, CC.9-12.F.BF.1***

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COMMON CORE

8-7

Solving Quadratic Equations by Using Square Roots

CC.9-12.A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, … *Also* **CC.9-12.A.CED.3*, CC.9-12.A.CED.1*, CC.9-12.F.BF.1***

Objective
Solve quadratic equations by using square roots.

Why learn this?
Square roots can be used to find how much fencing is needed for a pen at a zoo. (See Example 4.)

Some quadratic equations cannot be easily solved by factoring. Square roots can be used to solve some of these quadratic equations. Recall from Lesson 1-5 that every positive real number has two square roots, one positive and one negative.

$3(3) = 3^2 = 9 \rightarrow \sqrt{9} = 3$ ← Positive square root of 9

$(-3)(-3) = (-3)^2 = 9 \rightarrow -\sqrt{9} = -3$ ← Negative square root of 9

Reading Math
The expression ± 3 is read "plus or minus three."

When you take the square root of a positive real number and the sign of the square root is not indicated, you must find both the positive and negative square root. This is indicated by $\pm\sqrt{\ }$.

$\pm\sqrt{9} = \pm 3$ ← Positive and negative square roots of 9

Know it! Note — Square-Root Property

WORDS	NUMBERS	ALGEBRA
To solve a quadratic equation in the form $x^2 = a$, take the square root of both sides.	$x^2 = 15$ $x = \pm\sqrt{15}$	If $x^2 = a$ and a is a positive real number, then $x = \pm\sqrt{a}$.

EXAMPLE 1 **Using Square Roots to Solve $x^2 = a$**

Solve using square roots.

A $x^2 = 16$

$x = \pm\sqrt{16}$ *Solve for x by taking the square root of both sides. Use ± to show both square roots.*

$x = \pm 4$

The solutions are 4 and −4.

Check

$x^2 = 16$	
$(4)^2$	16
16	16 ✓

Substitute 4 and −4 into the original equation.

$x^2 = 16$	
$(-4)^2$	16
16	16 ✓

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1 Introduce

EXPLORATION

Solving Quadratic Equations by Using Square Roots

In this activity, you will use what you already know about squares to solve some simple quadratic equations.

1. Complete the table.

x	1	2	3	4	5	6	7	8	9	10
x^2	1	4	9	16	25	36	49	64	81	100

2. Complete the table.

x	−1	−2	−3	−4	−5	−6	−7	−8	−9	−10
x^2	1	4	9	16	25	36	49	64	81	100

3. Suppose $x^2 = 36$. What can you say about the value of x? $x = \pm 6$
4. Suppose $x^2 = 100$. What can you say about the value of x? $x = \pm 10$
5. In general, if a is a positive real number, how many solutions are there to the equation $x^2 = a$? 2

THINK AND DISCUSS

6. **Explain** how to find the solutions of $x^2 = 144$. Find values of x that equal 144 when squared.
7. **Describe** how you can use your tables from Problems 1 and 2 to estimate the solutions of the equation $x^2 = 20$. There is one solution between 4 and 5 and one solution between −4 and −5.

Motivate

Have students translate the following sentence into an algebraic equation: The sum of the square of a number and ten is forty-six. $x^2 + 10 = 46$

Have students use guess-and-check to find the values of x that make the sentence true. 6 and −6

Tell students that in this lesson they will learn algebraic methods for solving these types of equations.

Explorations and answers are provided in your online resources.

Solve using square roots.

B $x^2 = -4$

$x = \pm\sqrt{-4}$ *There is no real number whose square is negative.*

There is no real solution.

CHECK IT OUT! **Solve using square roots. Check your answer.**

1a. $x^2 = 121$ ± 11 **1b.** $x^2 = 0$ 0 **1c.** $x^2 = -16$ no real solutions

If necessary, use inverse operations to isolate the squared part of a quadratic equation before taking the square root of both sides.

EXAMPLE 2 Using Square Roots to Solve Quadratic Equations

Solve using square roots.

Helpful Hint

0 is neither positive nor negative.

A $x^2 + 5 = 5$

$x^2 + 5 = 5$

$-5 \quad -5$ *Subtract 5 from both sides.*

$x^2 = 0$

$x = \pm\sqrt{0} = 0$ *Take the square root of both sides.*

The solution is 0.

B $4x^2 - 25 = 0$

$4x^2 - 25 = 0$

$+25 \quad +25$ *Add 25 to both sides.*

$\frac{4x^2}{4} = \frac{25}{4}$ *Divide both sides by 4.*

$x^2 = \frac{25}{4}$

$x = \pm\sqrt{\frac{25}{4}} = \pm\frac{5}{2}$ *Take the square root of both sides. Use ± to show both square roots.*

The solutions are $\frac{5}{2}$ and $-\frac{5}{2}$.

C $(x + 2)^2 = 9$

$(x + 2)^2 = 9$

$x + 2 = \pm\sqrt{9}$ *Take the square root of both sides. Use ± to show both square roots.*

$x + 2 = \pm 3$

$x + 2 = 3$ or $x + 2 = -3$ *Write two equations, using both the positive and negative square roots, and solve each equation.*

$-2 \quad -2 \quad -2 \quad -2$

$x = 1$ or $x = -5$

The solutions are 1 and −5.

Check

$(x + 2)^2 = 9$	
$(1 + 2)^2$	9
3^2	9
9	9 ✓

$(x + 2)^2 = 9$	
$(-5 + 2)^2$	9
$(-3)^2$	9
9	9 ✓

CHECK IT OUT! **Solve by using square roots. Check your answer.**

2a. $100x^2 + 49 = 0$ no real solutions **2b.** $(x - 5)^2 = 16$ 9, 1

When solving quadratic equations by using square roots, you may need to find the square root of a number that is not a perfect square. In this case, the answer is an irrational number. You can approximate the solutions.

COMMON ERROR ALERT

For equations like $4x^2 = 25$, students sometimes mentally stack the smaller number on top of the larger number and get $x^2 = \frac{4}{25}$. Remind students that the goal is to isolate x^2, so both sides will be divided by the coefficient of x^2.

Power Presentations with PowerPoint®

Additional Examples

Example 1

Solve using square roots.

A. $x^2 = 169$ ± 13

B. $x^2 = -49$ no real solutions

Example 2

Solve using square roots. Check your answer.

A. $x^2 + 7 = 7$ 0

B. $16x^2 - 49 = 0$ $\pm\frac{7}{4}$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLES 1–2

- Are there any real numbers whose square is negative?
- When does a quadratic equation of the form $x^2 = a$ have one solution?
- How do you take the square root of a fraction?

2 Teach

Guided Instruction

Discuss with students that squaring and finding square roots are inverse operations and can be used to isolate a variable in an equation. Remind students that if you take the square root of one side of the equation, you must also take the square root of the other side to maintain the balance.

Reaching All Learners

Through Visual Cues

Encourage students to circle on their own paper what is to be isolated in each step of solving the equations in this lesson.

1. Circle the entire x^2-term and think of how it can be isolated.
2. Circle only x^2 and think of how it can be isolated.
3. Circle the x in x^2, showing that the last operation to be undone is the square.

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Additional Examples

Example 3

Solve. Round to the nearest hundredth.

A. $x^2 = 15$ ± 3.87

B. $-3x^2 + 90 = 0$ ± 5.48

Example 4

Ms. Pirzada is building a retaining wall along one of the long sides of her rectangular garden. The garden is twice as long as it is wide. It also has an area of 578 square feet. What will be the length of the retaining wall? 34 ft

Also available online

Questioning Strategies

EXAMPLE 3

- Describe a method to approximate the solution without a calculator.

EXAMPLE 4

- Why is only one of the solutions to the equation used to answer the question?

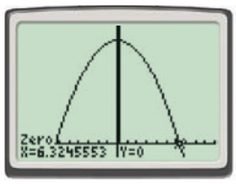
Teaching Tip **Critical Thinking** Ask students what is wrong with solving the equation $x^2 = 3x$ by dividing both sides by x. If they divide by x, the equation is $x = 3$, and they have lost the solution $x = 0$. Also, be sure they realize that if $x = 0$, division of both sides by x is undefined.

EXAMPLE 3 **Solve the equation $0 = -2x^2 + 80$. Round to the nearest hundredth.**

$$0 = -2x^2 + 80$$
$$-80 \quad -80 \qquad \text{Subtract 80 from both sides.}$$
$$\frac{-80}{-2} = \frac{-2x^2}{-2} \qquad \text{Divide both sides by } -2.$$
$$40 = x^2$$
$$\pm\sqrt{40} = x \qquad \text{Take the square root of both sides.}$$
$$x \approx \pm 6.32 \qquad \text{Find } \sqrt{40} \text{ on a calculator.}$$

The approximate solutions are 6.32 and −6.32.

Check Use a graphing calculator to support your answer.



Use the zero function.
The approximate solutions are 6.32 and −6.32. ✓

CHECK IT OUT! **Solve. Round to the nearest hundredth.**

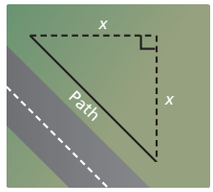
3a. $0 = 90 - x^2$ ± 9.49

3b. $2x^2 - 64 = 0$ ± 5.66

3c. $x^2 + 45 = 0$ no real solutions

EXAMPLE 4 ***Career Application***

A zookeeper is buying fencing to enclose a pen at the zoo. The pen is an isosceles right triangle. There is already a fence on the side that borders a path. The area of the pen will be 4500 square feet. The zookeeper can buy the fencing in whole feet only. How many feet of fencing should he buy?



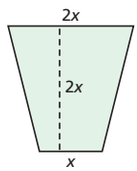
Let x represent the length of one of the sides.

Remember! An isosceles triangle has at least two sides of the same length.

$$\frac{1}{2}bh = A \qquad \text{Use the formula for area of a triangle.}$$
$$\frac{1}{2}x(x) = 4500 \qquad \text{Substitute } x \text{ for both } b \text{ and } h \text{ and } 4500 \text{ for } A.$$
$$(2)\frac{1}{2}x^2 = 4500(2) \qquad \text{Simplify. Multiply both sides by 2.}$$
$$x^2 = 9000$$
$$x = \pm\sqrt{9000} \qquad \text{Take the square root of both sides.}$$
$$x \approx \pm 94.9 \qquad \text{Find } \sqrt{9000} \text{ on a calculator.}$$

Negative numbers are not reasonable for length, so $x \approx 94.9$ is the only solution that makes sense. Therefore, the zookeeper needs 95 + 95, or 190, feet of fencing.

CHECK IT OUT! **4.** A house is on a lot that is shaped like a trapezoid. The solid lines show the boundaries, where x represents the width of the front yard. Find the width of the front yard, given that the area is 6000 square feet. Round to the nearest foot. (*Hint:* Use $A = \frac{1}{2}h(b_1 + b_2)$.) 45 ft



3 Close

Summarize

Review the steps for solving quadratic equations, by using square roots, comparing them to steps for solving linear equations: undo the operations performed on the variable. For quadratic equations, the last step is to take the square root of each side.

Have students state the steps they would use to solve $5x^2 - 18 = 27$.

Add 18 to each side. Divide both sides by 5. Take the square root of both sides.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

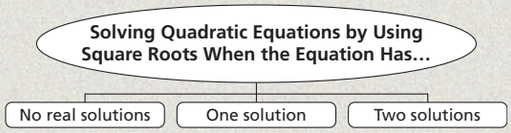
***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Explain why there are no solutions to the quadratic equation $x^2 = -9$.
2. Describe how to estimate the solutions of $4 = x^2 - 16$. What are the approximate solutions?
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of a quadratic equation with the given number of solutions. Solve each equation.

Know it! Note



8-7 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

Solve using square roots. Check your answer.

SEE EXAMPLE 1

1. $x^2 = 225$ ±15
2. $x^2 = 49$ ±7
3. $x^2 = -100$ no real solutions
4. $x^2 = 400$ ±20
5. $-25 = x^2$ no real solutions
6. $36 = x^2$ ±6

SEE EXAMPLE 2

7. $3x^2 - 75 = 0$ ±5
8. $0 = 81x^2 - 25$ $\pm\frac{5}{9}$
9. $49x^2 + 64 = 0$ no real solutions
10. $16x^2 + 10 = 131$ $\pm\frac{11}{4}$
11. $(x - 3)^2 = 64$ 11, −5
12. $(x - 9)^2 = 25$ 4, 14

SEE EXAMPLE 3

Solve. Round to the nearest hundredth.

13. $3x^2 = 81$ ±5.20
14. $0 = x^2 - 60$ ±7.75
15. $100 - 5x^2 = 0$ ±4.47

SEE EXAMPLE 4

16. **Geometry** The length of a rectangle is 3 times its width. The area of the rectangle is 170 square meters. Find the width. Round to the nearest tenth of a meter. (*Hint:* Use $A = \ell w$.) 7.5 m

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
17–22	1
23–28	2
29–34	3
35	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Solve using square roots. Check your answer.

17. $x^2 = 169$ ±13
18. $x^2 = 25$ ±5
19. $x^2 = -36$ no real solutions
20. $x^2 = 10{,}000$ ±100
21. $-121 = x^2$ no real solutions
22. $625 = x^2$ ±25
23. $4 - 81x^2 = 0$ $\pm\frac{2}{9}$
24. $(2x + 1)^2 = 25$ 2, −3
25. $64x^2 - 5 = 20$ $\pm\frac{5}{8}$
26. $(x - 7)^2 = 1$ 6, 8
27. $49x^2 + 1 = 170$ $\pm\frac{13}{7}$
28. $81x^2 + 17 = 81$ $\pm\frac{8}{9}$

Solve. Round to the nearest hundredth.

29. $4x^2 = 88$ ±4.69
30. $x^2 - 29 = 0$ ±5.39
31. $x^2 + 40 = 144$ ±10.20
32. $3x^2 - 84 = 0$ ±5.29
33. $50 - x^2 = 0$ ±7.07
34. $2x^2 - 10 = 64$ ±6.08

Teacher to Teacher

Throughout the lesson and as students work through the exercises, it is important to remind them that the graph of the related function can be used to confirm each of their answers. This reminds students again of the relationship between the solutions of an algebraic equation, the zeros of the related function, and the points containing the *x*-intercepts of the function's graph.

Jill Springer
Henderson, KY

Answers to *Think and Discuss*

Possible answers:

1. There is no real number whose square is negative.
2. The equation is equivalent to $x^2 = 20$, so $x = \pm\sqrt{20}$. $\sqrt{20}$ is between 4 and 5, so x is approximately ±4.5.
3. See Additional Answers.

8-7 Exercises

Assignment Guide

Assign *Guided Practice* Exercises as necessary.

If you finished Examples **1–2**
Basic 17–28, 43, 53
Average 17–28, 43, 44, 53
Advanced 17–28, 39, 44, 53

If you finished Examples **1–4**
Basic 17–35, 42, 43, 46–49, 54–56
Average 17–38, 40–52, 54–58
Advanced 17–42, 44–60

Homework Quick Check
Quickly check key concepts.
Exercises: 22, 24, 32, 35, 46

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 16, 35, 40, 42, 46, 60

Reason abstractly and quantitatively. Exercises 39, 44–45

Construct viable arguments and critique the reasoning of others. Exercises 43, 47, 53

Attend to precision. Exercises 41

Learn It Online
State Resources Online

Teaching Tip **Geometry** In **Exercise 40,** students find a geometric mean. The geometric mean of two positive numbers a and b can be thought of the side length of a square whose area is equal to that of a rectangle with side lengths a and b. In other words, the geometric mean of a and b is $\sqrt{ab}$.

Multi-Step Test Prep **Exercise 48** involves solving a quadratic equation related to the height of a falling golf ball over time. This exercise prepares students for the Multi-Step Test Pre.

35. Entertainment For a scene in a movie, a sack of money is dropped from the roof of a 600 ft skyscraper. The height of the sack above the ground is given by $h = -16t^2 + 600$, where t is the time in seconds. How long will it take the sack to reach the ground? Round to the nearest tenth of a second. **6.1 s**

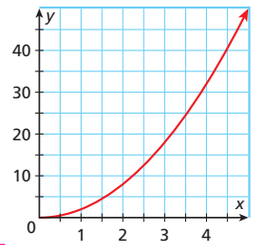
Solve for the indicated variable. Assume all values are positive.

36. $A = \pi r^2$ for r $r = \sqrt{\frac{A}{\pi}}$ **37.** $d = \frac{1}{2}at^2$ for t $t = \sqrt{\frac{2d}{a}}$ **38.** $F = \frac{mv^2}{r}$ for v $v = \sqrt{\frac{Fr}{m}}$

39. Number Theory If $a = 2b$ and $2ab = 36$, find all possible solutions for a and b. **$a = -6$ and $b = -3$ or $a = 6$ and $b = 3$**

40. Geometry The geometric mean of two positive numbers a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$. Find the geometric mean of 2 and 18. **6**

41. Estimation The area y of any rectangle with side length x and one side twice as long as the other is represented by $y = 2x^2$. Use the graph to estimate the dimensions of such a rectangle whose area is 35 square feet. **about 4.2 ft by 8.4 ft**



42. Physics The period of a pendulum is the amount of time it takes to swing back and forth one time. The relationship between the length of the pendulum L in inches and the length of the period t in seconds can be approximated by $L = 9.78t^2$. Find the period of a pendulum whose length is 60 inches. Round to the nearest tenth of a second. **about 2.5 s**

43. ERROR ANALYSIS Which solution is incorrect? Explain the error.

A

$x^2 + 100 = 0$
$x^2 = 100$
$x = 10$ or $x = -10$

B

$x^2 + 100 = 0$
$x^2 = -100$
no solution

A; 100 was added to the right side of the equation instead of subtracted.

Determine whether each statement is always, sometimes, or never true.

44. There are two solutions to $x^2 = n$ when n is positive. **always**

45. If n is a rational number, then the solutions to $x^2 = n$ are rational numbers. **sometimes**

46. Multi-Step The height in feet of a soccer ball kicked upward from the ground with initial velocity 60 feet per second is modeled by $h = -16t^2 + 60t$, where t is the time in seconds. Find the time it takes for the ball to return to the ground. **3.75 s**

47. Critical Thinking For the equation $x^2 = a$, describe the values of a that will result in each of the following.

a. two solutions **$a > 0$**

b. one solution **$a = 0$**

c. no solution **$a < 0$**

Multi-Step Test Prep

48. The equation $d = 16t^2$ describes the distance d in feet that a golf ball falls in relation to the number of seconds t that it falls.

a. How many seconds will it take a golf ball to drop to the ground from a height of 4 feet? **$\frac{1}{2}$ s**

b. Make a table and graph the function.

c. How far will the golf ball drop in 1 second? **16 ft**

d. How many seconds will it take the golf ball to drop 64 feet? **2 s**

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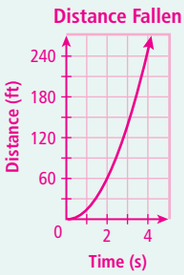
Answers

48b.

t	$d = 16t^2$
0	0
1	16
2	64
3	144
4	256



PRACTICE A

PRACTICE C

PRACTICE B

Name _____ Date _____ Class _____

LESSON 8-7 **Practice B**

Solving Quadratic Equations by Using Square Roots

Solve using square roots. Check your answer.

1. $x^2 = 81$
   $x = \pm\sqrt{81}$
   $x = \pm 9$
   The solutions are 9 and −9.
2. $x^2 = 100$
   $x = \pm\sqrt{100}$
   $x = \pm 10$
   The solutions are 10 and −10.
3. $x^2 = 225$
   $x = \pm\sqrt{225}$
   $x = \pm 15$
4. $441 = x^2$
   $\pm\sqrt{441} = x$
   $\pm 21 = x$
5. $x^2 = -400$
   no real solutions
6. $3x^2 = 108$
   $-4, 8$
7. $100 = 4x^2$
   ± 5
8. $x^2 + 7 = 71$
   ± 8
9. $49x^2 - 64 = 0$
   $\pm\frac{8}{7}$
10. $-2x^2 = -162$
    ± 9
11. $9x^2 + 100 = 0$
    no real solutions
12. $0 = 81x^2 - 121$
    $\pm\frac{11}{9}$
13. $100x^2 = 25$
    $\pm\frac{1}{2}$
14. $100x^2 = 121$
    $-4, 8$

Solve. Round to the nearest hundredth.

15. $8x^2 = 56$
    ± 2.65
16. $5 - x^2 = 20$
    no real solutions
17. $x^2 + 35 = 105$
    ± 8.37
18. The height of a skydiver jumping out of an airplane is given by $h = -16t^2 + 3200$. How long will it take the skydiver to reach the ground? Round to the nearest tenth of a second.
    14.1 seconds
19. The height of a triangle is twice the length of its base. The area of the triangle is 50 m². Find the height and base to the nearest tenth of a meter.
    height: 14.2 m, base: 7.1 m
20. The height of an acorn falling out of a tree is given by $h = -16t^2 + b$. If an acorn takes 1 second to fall to the ground. What is the value of b?
    $b = 16$

READING STRATEGIES

Name _____ Date _____ Class _____

LESSON 8-7 **Reading Strategies**

Making Connections

You've learned to solve quadratic equations by graphing. Equations that are in the form $ax^2 + c = 0$ (where $b = 0$) can also be solved by using square roots. Study the examples below to make the connection between solving quadratic equations by graphing and solving by using square roots.

Example 1: The equation $x^2 = 9$ has **2 solutions.**

Solve by using square roots:	Solve by graphing:
$x^2 = 9$	$x^2 = 9$
$x = \pm\sqrt{9}$	$x^2 - 9 = 0$
$x = \pm 3$	$x^2 - 9 = y$
$x = 3$ and $x = -3$	$y = x^2 - 9$
The solutions are 3 and −3.	The graph has zeros at 3 and −3.

Example 2: The equation $x^2 = -9$ has **no real solutions.**

Solve by using square roots:	Solve by graphing:
$x^2 = -9$	$x^2 = -9$
$x = \pm\sqrt{-9}$	$x^2 + 9 = 0$
There is no real number whose square is negative.	$x^2 + 9 = y$
There are no solutions.	$y = x^2 + 9$
	The graph does not cross the *x*-axis

Example 3: The equation $x^2 + 9 = 9$ has **1 solution.**

Solve by using square roots:	Solve by graphing:
$x^2 + 9 = 9$	$x^2 + 9 = 9$
$x^2 = 0$	$x^2 + 0 = 0$
$x = \pm\sqrt{0}$	$x^2 = x$
$x = 0$	$y = x^2$
The solutions is 0.	The graph has a zero at 0.

Complete the following.

1. Solve $x^2 = 25$ by using square roots. $x = \pm 5$
2. Sketch a graph of the related function $y = x^2 - 25$ to show that you get the same solution(s).
3. How many solutions does $x^2 = -36$ have? zero
   a. Explain, in terms of the graph: The parabola

RETEACH

Name _____ Date _____ Class _____

LESSON 8-7 **Review for Mastery**

Solving Quadratic Equations by Using Square Roots

If a quadratic equation is in the form $x^2 = a$, you must take the square root of both sides to find the solutions. Remember to find both the positive and negative square roots.

Solve $x^2 = 36$ using square roots.

		Check: $x = 6$	Check: $x = -6$
$x^2 = 36$			
$\sqrt{x^2} = \pm\sqrt{36}$	*Take the square root of both sides.*	$x^2 = 36$	$x^2 = 36$
$x = \pm 6$		$(6)^2 \stackrel{?}{=} 36$	$(-6)^2 \stackrel{?}{=} 36$
The solutions are 6 and −6.		$36 \stackrel{?}{=} 36$ ✓	$36 \stackrel{?}{=} 36$ ✓

Solve $2x^2 + 7 = 207$ using square roots.

		Check: $x = 10$	Check: $x = -10$
$2x^2 + 7 = 207$			
$\frac{-7}{2x^2} = \frac{-7}{200}$	*Add −7 to both sides.*	$2x^2 + 7 \stackrel{?}{=} 207$	$2x^2 + 7 \stackrel{?}{=} 207$
$\frac{2x^2}{2} = \frac{200}{2}$	*Divide both sides by 2.*	$2(10)^2 + 7 \stackrel{?}{=} 207$	$2(-10)^2 + 7 \stackrel{?}{=} 207$
$\sqrt{x^2} = \pm\sqrt{100}$	*Take the square root of both sides.*	$2(100) + 7 \stackrel{?}{=} 207$	$2(100) + 7 \stackrel{?}{=} 207$
$x = \pm 10$		$200 + 7 \stackrel{?}{=} 207$	$200 + 7 \stackrel{?}{=} 207$
The solutions are 10 and −10.		$207 \stackrel{?}{=} 207$ ✓	$207 \stackrel{?}{=} 207$ ✓

Solve using square roots.

1. $x^2 = 81$
   ±9
2. $x^2 = 9$
   ±3
3. $x^2 = -64$
   no solution
4. $x^2 + 44 = 188$
   ±12
5. $x^2 - 12 = 37$
   ±7
6. $x^2 + 10 = 131$
   ±11
7. $3x^2 + 25 = 73$
   ±4
8. $5x^2 - 9 = 116$
   ±5
9. $-4x^2 + 42 = -102$
   ±6
10. $4x^2 - 11 = 25$
    ±3
11. $x^2 - 13 = 87$
    ±10
12. $-3x^2 + 200 = 8$
    ±8

49. no; $x = \pm\sqrt{\frac{1}{2}}$, irrational

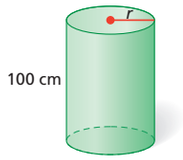
For the quadratic equation $x^2 + a = 0$, determine whether each value of a will result in two rational solutions. Explain.

49. $-\frac{1}{2}$ **50.** $\frac{1}{2}$ **51.** $-\frac{1}{4}$ **52.** $\frac{1}{4}$

53. Write About It Explain why the quadratic equation $x^2 + 4 = 0$ has no real solutions but the quadratic equation $x^2 - 4 = 0$ has two solutions.

TEST PREP

54. The formula for finding the approximate volume of a cylinder is $V = 3.14r^2h$, where r is the radius and h is the height. The height of a cylinder is 100 cm, and the approximate volume is 1256 cm^3. Find the radius of the cylinder.



(A) 400 cm (C) 4 cm
(B) 20 cm (D) 2 cm

55. Which best describes the positive solution of $\frac{1}{2}x^2 = 20$?

(F) Between 4 and 5 (H) Between 6 and 7
(G) Between 5 and 6 (J) Between 7 and 8

56. Which best describes the solutions of $81x^2 - 169 = 0$?

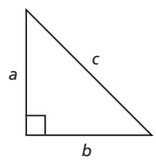
(A) Two rational solutions (C) No solution
(B) Two irrational solutions (D) One solution

CHALLENGE AND EXTEND

Find the solutions of each equation without using a calculator.

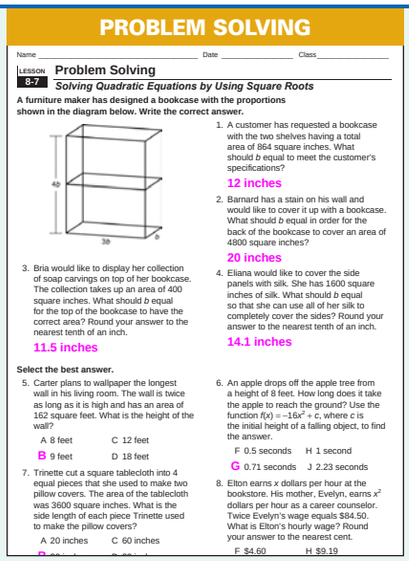
57. $288x^2 - 19 = -1$ $\pm\frac{1}{4}$ **58.** $-75x^2 = -48$ $\pm\frac{4}{5}$ **59.** $x^2 = \frac{128}{242}$ $\pm\frac{8}{11}$

60. Geometry The Pythagorean Theorem states that $a^2 + b^2 = c^2$ if a and b represent the lengths of the legs of a right triangle and c represents the length of the hypotenuse.



a. Find the length of the hypotenuse if the lengths of the legs are 9 cm and 12 cm. 15 cm

b. Find the length of each leg of an isosceles right triangle whose hypotenuse is 10 cm. Round to the nearest tenth of a centimeter. 7.1 cm


PROBLEM SOLVING

Name _____ Date _____ Class _____

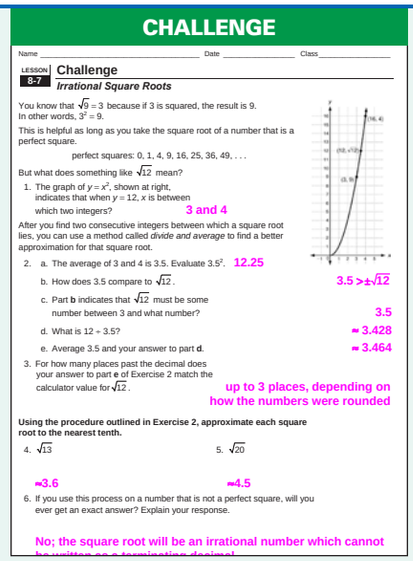
LESSON 8-7 Problem Solving
Solving Quadratic Equations by Using Square Roots

A furniture maker has designed a bookcase with the proportions shown in the diagram below. Write the correct answer.

1. A customer has requested a bookcase with the two shelves having a total area of 864 square inches. What should b equal to meet the customer's specifications?
12 inches
2. Barnard has a stain on his wall and would like to cover it up with a bookcase. What should b equal in order for the back of the bookcase to cover an area of 4800 square inches?
20 inches
3. Bria would like to display her collection of soap carvings on top of her bookcase. The collection takes up an area of 400 square inches. What should b equal for the top of the bookcase to have the correct area? Round your answer to the nearest tenth of an inch.
11.5 inches
4. Eliana would like to cover the side panels with silk. She has 1600 square inches of silk. What should b equal so that she can use all of her silk to completely cover the sides? Round your answer to the nearest tenth of an inch.
14.1 inches

Select the best answer.

5. Carter plans to wallpaper the longest wall in his living room. The wall is twice as long as it is high and has an area of 162 square feet. What is the height of the wall?
A 8 feet C 12 feet
B 9 feet D 18 feet
6. An apple drops off the apple tree from a height of 8 feet. How long does it take the apple to reach the ground? Use the function $f(x) = -16x^2 + c$, where c is the initial height of a falling object, to find the answer.
F 0.5 seconds H 1 second
G 0.71 seconds J 2.23 seconds
7. Trinette cut a square tablecloth into 4 equal pieces that she used to make two pillow covers. The area of the tablecloth was 3600 square inches. What is the side length of each piece Trinette used to make the pillow covers?
A 20 inches C 60 inches
8. Elton earns x dollars per hour at the bookstore. His mother, Evelyn, earns x^2 dollars per hour as a career counselor. Twice Evelyn's wage equals $84.50. What is Elton's hourly wage? Round your answer to the nearest cent.
F $4.60 H $9.19


CHALLENGE

Name _____ Date _____ Class _____

LESSON 8-7 Challenge
Irrational Square Roots

You know that $\sqrt{9} = 3$ because if 3 is squared, the result is 9. In other words, $3^2 = 9$.

This is helpful as long as you take the square root of a number that is a perfect square.

perfect squares: 0, 1, 4, 9, 16, 25, 36, 49, . . .

But what does something like $\sqrt{12}$ mean?

1. The graph of $y = x^2$, shown at right, indicates that when $y = 12$, x is between which two integers? 3 and 4

After you find two consecutive integers between which a square root lies, you can use a method called *divide and average* to find a better approximation for that square root.

2. a. The average of 3 and 4 is 3.5. Evaluate 3.5^2. 12.25
b. How does 3.5 compare to $\sqrt{12}$? $3.5 > \pm\sqrt{12}$
c. Part **b** indicates that $\sqrt{12}$ must be some number between 3 and what number? 3.5
d. What is $12 \div 3.5$? ≈ 3.428
e. Average 3.5 and your answer to part **d**. ≈ 3.464
3. For how many places past the decimal does your answer to part **e** of Exercise 2 match the calculator value for $\sqrt{12}$? up to 3 places, depending on how the numbers were rounded

Using the procedure outlined in Exercise 2, approximate each square root to the nearest tenth.

4. $\sqrt{13}$ ≈ 3.6
5. $\sqrt{20}$ ≈ 4.5
6. If you use this process on a number that is not a perfect square, will you ever get an exact answer? Explain your response.

No; the square root will be an irrational number which cannot

TEST PREP DOCTOR For **Exercise 56,** remind students that rational numbers are numbers that can be written in the form $\frac{a}{b}$, where a and b are both integers and b is not zero. For students who change the fraction to a decimal, remind them that repeating decimals are also rational numbers.

Answers

50. no; $x = \pm\sqrt{-\frac{1}{2}}$, no real solutions
51. yes; $x = \pm\frac{1}{2}$, rational
52. no; $x = \pm\sqrt{-\frac{1}{4}}$, no real solutions
53. See Additional Answers.

Journal

Have students explain how to solve an equation of the form $ax^2 + c = 0$ by using square roots.

ALTERNATIVE ASSESSMENT

Have students create and solve quadratic equations in the form $ax^2 + c = 0$ where 1) a and c are both positive, 2) a and c are both negative, and 3) a is positive and c is negative.

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8-7 Lesson Quiz

Solve using square roots. Check your answer.

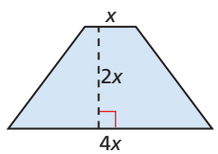
1. $x^2 - 195 = 1$ ± 14

2. $4x^2 - 18 = -9$ $\pm\frac{3}{2}$

3. $(x + 7)^2 = 81$ $-16, 2$

4. Solve $0 = -5x^2 + 225$. Round to the nearest hundredth. ± 6.71

5. A community swimming pool is in the shape of a trapezoid. The height of the trapezoid is twice as long as the shorter base and the longer base is twice as long as the height.



The area of the pool is 3675 square feet. What is the length of the longer base? Round to the nearest foot. (*Hint:* Use $A = \frac{1}{2}h(b_1 + b_2)$.) 108 feet

Also available online

Organizer

Use with the lesson *Completing the Square*

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use algebra tiles to model completing the square.

Materials: algebra tiles

Online Edition

Countdown

Resources

Algebra Lab Activities
Lab Recording Sheet

Teach

Discuss

In the second step of the activity, discuss why 9 unit-tiles are added to the mat. Explain that the goal is to make a perfect square by adding one-tiles.

Close

Key Concept

You can use algebra tiles to model completing the square to form a perfect-square trinomial. Algebra tiles can be found in the Manipulatives Kit (MK). Note that completing the square results in an expression that is not equivalent to the original expression.

Assessment

Journal Have students explain how to use algebra tiles to complete the square for $x^2 + 20x$. Have them write the new expression in factored form. $x^2 + 20x + 100$; $(x + 10)^2$

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.A.REI.4 Solve quadratic equations in one variable.
a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions….

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State Resources Online

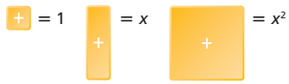
8-8

Model Completing the Square

Use with Completing the Square

One way to solve a quadratic equation is by using a procedure called *completing the square*. In this procedure, you add something to a quadratic expression to make it a perfect-square trinomial. This procedure can be modeled with algebra tiles.

KEY

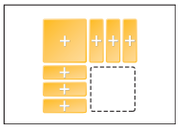
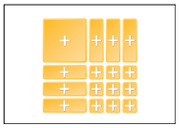
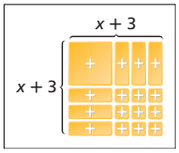


MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

CC.9-12.A.REI. 4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions….

Activity

Use algebra tiles to model $x^2 + 6x$. Add unit tiles to complete a perfect-square trinomial. Then write the new expression in factored form.

MODEL		ALGEBRA
	*Arrange the tiles to form part of a large square.* *Part of the square is missing. How many one-tiles do you need to complete it?*	$x^2 + 6x$
	*Complete the square by placing 9 one-tiles on the mat.* *$x^2 + 6x + 9$ is a perfect-square trinomial.*	$x^2 + 6x + 9$
	*Use the length and the width of the square to rewrite the area expression in factored form.*	$(x + 3)^2$

Try This

Use algebra tiles to model each expression. Add unit tiles to complete a perfect-square trinomial. Then write the new expression in factored form.

1. $x^2 + 4x$
2. $x^2 + 2x$
3. $x^2 + 10x$
4. $x^2 + 8x$
5. **Make a Conjecture** Examine the pattern in Problems 1–4. How many unit tiles would you have to add to make $x^2 + 12x$ a perfect-square trinomial? **36**

Answers to *Try This*

1. $x^2 + 4x + 4 = (x + 2)^2$
2. $x^2 + 2x + 1 = (x + 1)^2$
3. $x^2 + 10x + 25 = (x + 5)^2$
4. $x^2 + 8x + 16 = (x + 4)^2$

COMMON CORE

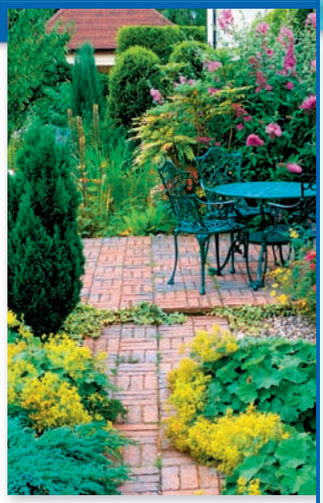
8-8 Completing the Square

CC.9-12.A.REI. 4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions…. *Also* **CC.9-12.A.CED.1***

Objective
Solve quadratic equations by completing the square.

Vocabulary
completing the square

Who uses this?
Landscapers can solve quadratic equations to find dimensions of patios. (See Example 4.)



Previously, you solved quadratic equations by isolating x^2 and then using square roots. This method works if the quadratic equation, when written in standard form, is a perfect square.

When a trinomial is a perfect square, there is a relationship between the **coefficient of the x-term** and the **constant term**.

$x^2 + 6x + 9$ $\quad$ $\left(\frac{6}{2}\right)^2 = 9$ $\qquad$ $x^2 - 8x + 16$ $\quad$ $\left(\frac{-8}{2}\right)^2 = 16$

Divide the coefficient of the x-term by 2, then square the result to get the constant term.

An expression in the form $x^2 + bx$ is not a perfect square. However, you can use the relationship shown above to add a term to $x^2 + bx$ to form a trinomial that is a perfect square. This is called **completing the square**.

Know it! Note

Completing the Square

WORDS	NUMBERS	ALGEBRA
To complete the square of $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$ to the expression. This will form a perfect-square trinomial.	$x^2 + 6x + \square$ $x^2 + 6x + \left(\frac{6}{2}\right)^2$ $x^2 + 6x + 9$ $(x + 3)^2$	$x^2 + bx + \square$ $x^2 + bx + \left(\frac{b}{2}\right)^2$ $\left(x + \frac{b}{2}\right)^2$

EXAMPLE 1 **Completing the Square**

Complete the square to form a perfect-square trinomial.

A $x^2 + 10x + \square$

$x^2 + 10x$

$\left(\frac{10}{2}\right)^2 = 5^2 = 25$ *Find $\left(\frac{b}{2}\right)^2$.*

$x^2 + 10x + 25$ *Add $\left(\frac{b}{2}\right)^2$ to the expression.*

Identify b.

B $x^2 - 9x + \square$

$x^2 + -9x$

$\left(\frac{-9}{2}\right)^2 = \frac{81}{4}$

$x^2 - 9x + \frac{81}{4}$

CHECK IT OUT!

Complete the square to form a perfect-square trinomial.

1a. $x^2 + 12x + 36$ **1b.** $x^2 - 5x + \frac{25}{4}$ **1c.** $8x + x^2 + 16$

To solve a quadratic equation in the form $x^2 + bx = c$, first complete the square of $x^2 + bx$. Then you can solve using square roots.

The Garden Picture Library / Alamy

1 Introduce

EXPLORATION

Completing the Square

Recall that you can square a binomial by using the following rule:

$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$.

1. Complete the table by squaring the binomials.

Binomial Squared	Trinomial
$(x + 2)^2$	$x^2 + 4x + 4$
$(x - 5)^2$	$x^2 - 10x + 25$
$(x + 3)^2$	$x^2 + 6x + 9$
$(x - 4)^2$	$x^2 - 8x + 16$
$(x - 7)^2$	$x^2 - 14x + 49$
$(x + 9)^2$	$x^2 + 18x + 81$

2. The trinomials in the table are perfect-square trinomials. In each of them, how is the constant term related to the coefficient of the x-term? Divide the coefficient of the x-term by 2 and square the result to get the constant term.
3. Suppose you know that $x^2 - 12x + \square$ is a perfect-square trinomial. What must be the value of the constant term? 36
4. What binomial was squared in order to get the perfect-square trinomial in Problem 3? $(x - 6)^2$

THINK AND DISCUSS

5. **Explain** how you found the constant term of the perfect-square trinomial in Problem 3. Divide −12 by 2 and square the result.

Motivate

Display $x^2 + 8x + \square$. Have students find four numbers that make the trinomial factorable.
Possible answer: 7, 12, 15, 16

Which makes it a perfect-square trinomial? 16

Explorations and answers are provided in your online resources.

8-8 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Solve quadratic equations by completing the square.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Simplify.

1. $\left(\frac{7}{2}\right)^2$ $\frac{49}{4}$
2. $\left(\frac{10}{2}\right)^2 - 6$ 19
3. $\left(\frac{-9}{2}\right)^2$ $\frac{81}{4}$
4. $\left(\frac{3}{2}\right)^2 + 3$ $\frac{21}{4}$

Solve each quadratic equation by factoring.

5. $x^2 + 8x + 16 = 0$ -4
6. $x^2 - 22x + 121 = 0$ 11
7. $x^2 - 12x + 36 = 0$ 6

Also available online

Math Humor

Q: What did the quadratic binomial say when he proposed to the constant?

A: You complete me.

COMMON CORE

CC.9-12.A.REI.4 Solve quadratic equations in one variable.
a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions….
CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems.*

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Additional Examples

Example 1

Complete the square to form a perfect-square trinomial.

A. $x^2 + 2x +$ ▢ 1

B. $x^2 - 6x +$ ▢ 9

Example 2

Solve by completing the square. Check your answer.

A. $x^2 + 16x = -15$ $-1, -15$

B. $x^2 - 4x - 6 = 0$ $2 \pm \sqrt{10}$

Also available online

Questioning Strategies

EXAMPLE 1

- Can the number added to complete the square ever be negative? Explain.

EXAMPLE 2

- In what form should the equation be before you find $\left(\frac{b}{2}\right)^2$?
- When will solutions be irrational? Give an example.

Teaching Tip **Math Background** When the solution of a quadratic equation involves the square root of a negative number, we say it has no real solutions. In fact, these types of quadratic equations have nonreal solutions, called *complex numbers*. Students will study complex numbers in Algebra 2.

Know it! Note

Solving a Quadratic Equation by Completing the Square
Step 1 Write the equation in the form $x^2 + bx = c$.
Step 2 Find $\left(\frac{b}{2}\right)^2$.
Step 3 Complete the square by adding $\left(\frac{b}{2}\right)^2$ to both sides of the equation.
Step 4 Factor the perfect-square trinomial.
Step 5 Take the square root of both sides.
Step 6 Write two equations, using both the positive and negative square root, and solve each equation.

EXAMPLE 2 Solving $x^2 + bx = c$ by Completing the Square

Solve by completing the square. Check your answer.

A $x^2 + 14x = 15$

Step 1 $x^2 + 14x = 15$ *The equation is in the form $x^2 + bx = c$.*

Step 2 $\left(\frac{14}{2}\right)^2 = 7^2 = 49$ *Find $\left(\frac{b}{2}\right)^2$.*

Step 3 $x^2 + 14x + 49 = 15 + 49$ *Complete the square.*

Step 4 $(x + 7)^2 = 64$ *Factor and simplify.*

Step 5 $x + 7 = \pm 8$ *Take the square root of both sides.*

Step 6 $x + 7 = 8$ or $x + 7 = -8$ *Write and solve two equations.*
$x = 1$ or $x = -15$

Helpful Hint

$(x + 7)(x + 7) = (x + 7)^2$. So the square root of $(x + 7)^2$ is $x + 7$.

The solutions are 1 and -15.

Check

$x^2 + 14x = 15$	
$(1)^2 + 14(1)$	15
$1 + 14$	15
15	15 ✓

$x^2 + 14x = 15$	
$(-15)^2 + 14(-15)$	15
$225 - 210$	15
15	15 ✓

B $x^2 - 2x - 2 = 0$

Step 1 $x^2 + (-2)x = 2$ *Write in the form $x^2 + bx = c$.*

Step 2 $\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$ *Find $\left(\frac{b}{2}\right)^2$.*

Step 3 $x^2 - 2x + 1 = 2 + 1$ *Complete the square.*

Step 4 $(x - 1)^2 = 3$ *Factor and simplify.*

Step 5 $x - 1 = \pm\sqrt{3}$ *Take the square root of both sides.*

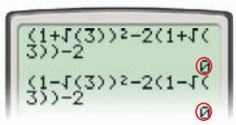
Step 6 $x - 1 = \sqrt{3}$ or $x - 1 = -\sqrt{3}$ *Write and solve two equations.*
$x = 1 + \sqrt{3}$ or $x = 1 - \sqrt{3}$

Writing Math

The expressions $1 + \sqrt{3}$ and $1 - \sqrt{3}$ can be written as one expression: $1 \pm \sqrt{3}$, which is read as "1 plus or minus the square root of 3."

The solutions are $1 + \sqrt{3}$ and $1 - \sqrt{3}$.

Check Use a graphing calculator to check your answer.



CHECK IT OUT! **Solve by completing the square. Check your answer.**

2a. $x^2 + 10x = -9$ $-9, -1$ **2b.** $t^2 - 8t - 5 = 0$ $4 \pm \sqrt{21}$

2 Teach

Guided Instruction

Review how to factor perfect-square trinomials and how to solve quadratic equations by using square roots. Stress that the equation must be in the form $x^2 + bx = c$ before $\left(\frac{b}{2}\right)^2$ is added to both sides.

Teaching Tip **Visual** Use algebra tiles (MK) to help students see how they are forming a perfect square.

Reaching All Learners *Through Communication*

Have students work in small groups to make a poster showing how to apply the six steps for solving quadratic equations by factoring. Give each group a different equation to solve. Then have each group present their posters to the rest of the class, explaining each step.

EXAMPLE 3 Solving $ax^2 + bx = c$ by Completing the Square

Solve by completing the square.

A $-2x^2 + 12x - 20 = 0$

Step 1 $\frac{-2x^2}{-2} + \frac{12x}{-2} - \frac{20}{-2} = \frac{0}{-2}$ *Divide by –2 to make a = 1.*

$x^2 - 6x + 10 = 0$ *Write in the form $x^2 + bx = c$.*

$x^2 - 6x = -10$

$x^2 + (-6)x = -10$

Step 2 $\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$ *Find $\left(\frac{b}{2}\right)^2$.*

Step 3 $x^2 - 6x + 9 = -10 + 9$ *Complete the square.*

Step 4 $(x - 3)^2 = -1$ *Factor and simplify.*

There is no real number whose square is negative, so there are no real solutions.

B $3x^2 - 10x = -3$

Step 1 $\frac{3x^2}{3} - \frac{10}{3}x = \frac{-3}{3}$ *Divide by 3 to make a = 1.*

$x^2 - \frac{10}{3}x = -1$

$x^2 + \left(-\frac{10}{3}\right)x = -1$ *Write in the form $x^2 + bx = c$.*

Step 2 $\left(-\frac{10}{3} \cdot \frac{1}{2}\right)^2 = \left(-\frac{10}{6}\right)^2 = \frac{100}{36} = \frac{25}{9}$ *Find $\left(\frac{b}{2}\right)^2$.*

Step 3 $x^2 - \frac{10}{3}x + \frac{25}{9} = -1 + \frac{25}{9}$ *Complete the square.*

$x^2 - \frac{10}{3}x + \frac{25}{9} = -\frac{9}{9} + \frac{25}{9}$ *Rewrite using like denominators.*

Step 4 $\left(x - \frac{5}{3}\right)^2 = \frac{16}{9}$ *Factor and simplify.*

Step 5 $x - \frac{5}{3} = \pm\frac{4}{3}$ *Take the square root of both sides.*

Step 6 $x - \frac{5}{3} = \frac{4}{3}$ or $x - \frac{5}{3} = -\frac{4}{3}$ *Write and solve two equations.*

$x = 3$ or $x = \frac{1}{3}$

The solutions are 3 and $\frac{1}{3}$.

Remember!
Dividing by 2 is the same as multiplying by $\frac{1}{2}$.

CHECK IT OUT! **Solve by completing the square.**

3a. $3x^2 - 5x - 2 = 0$ $-\frac{1}{3}, 2$

3b. $4t^2 - 4t + 9 = 0$ no real solutions

EXAMPLE 4 *Problem-Solving Application*

A landscaper is designing a rectangular brick patio. She has enough bricks to cover 144 square feet. She wants the length of the patio to be 10 feet greater than the width. What dimensions should she use for the patio?

1 Understand the Problem

The **answer** will be the length and width of the patio.

MATHEMATICAL PRACTICES
Make sense of problems and persevere in solving them.

COMMON ERROR ALERT

In **Example 3B,** students may have difficulty finding $\left(\frac{b}{2}\right)^2$ when b is a fraction. Remind them that dividing by 2 is the same as multiplying by $\frac{1}{2}$. For $b = -\frac{10}{3}$, $\left(\frac{b}{2}\right)^2$ becomes $\left[\frac{1}{2} \cdot \left(-\frac{10}{3}\right)\right]^2$, or $\left(\frac{10}{6}\right)^2$. Let students practice with a few examples and lead them to see that dividing a fraction by 2 is the same as doubling the denominator.

Power Presentations with PowerPoint®

Additional Examples

Example 3

Solve by completing the square.

A. $-3x^2 + 12x - 15 = 0$ no real solutions

B. $5x^2 + 19x = 4$ $-4, \frac{1}{5}$

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 3

- When will there be no real solutions?

Teaching Tip **Critical Thinking** Students can save time by checking that an equation has real solutions by first finding the vertex of the related function and using the value of a to see how the graph opens. In **Example 3A,** the related function has the vertex at $(3, -2)$, below the x-axis, and opens downward.

Teaching Tip **Inclusion** Remind students to always check their answers either by graphing the related function or by direct substitution.

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Additional Examples

Example 4

A rectangular room has an area of 195 square feet. Its width is 2 feet shorter than its length. Find the dimensions of the room.

13 ft by 15 ft

Also available online

Questioning Strategies

EXAMPLE 4

- How do you know which of the two solutions to use?
- How would the equation be different if the area were triangular?

Geometry

List the important information:

- There are enough bricks to cover 144 square feet.
- One edge of the patio is to be 10 feet longer than the other edge.

2 Make a Plan

Set the formula for the area of a rectangle equal to 144, the area of the patio. Solve the equation.

3 Solve

Let x be the width.

Then $x + 10$ is the length.

Use the formula for area of a rectangle.

$\ell \cdot w = A$

length times width = area of patio

$(x + 10) \cdot x = 144$

Step 1 $x^2 + 10x = 144$ *Simplify.*

Step 2 $\left(\frac{10}{2}\right)^2 = 5^2 = 25$ *Find* $\left(\frac{b}{2}\right)^2$.

Step 3 $x^2 + 10x + 25 = 144 + 25$ *Complete the square by adding 25 to both sides.*

Step 4 $(x + 5)^2 = 169$ *Factor the perfect-square trinomial.*

Step 5 $x + 5 = \pm 13$ *Take the square root of both sides.*

Step 6 $x + 5 = 13$ or $x + 5 = -13$ *Write and solve two equations.*

$x = 8$ or $x = -18$

Negative numbers are not reasonable for length, so $x = 8$ is the only solution that makes sense.

The width is 8 feet, and the length is $8 + 10$, or 18, feet.

4 Look Back

The length of the patio is 10 feet greater than the width. Also, $8(18) = 144$.

CHECK IT OUT!

4. An architect designs a rectangular room with an area of 400 ft^2. The length is to be 8 ft longer than the width. Find the dimensions of the room. Round your answers to the nearest tenth of a foot. **16.4 ft by 24.4 ft**

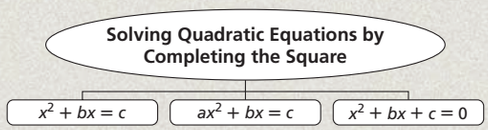
MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Tell how to solve a quadratic equation in the form $x^2 + bx + c = 0$ by completing the square.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write and solve an example of the given type of quadratic equation.



3 Close

Summarize

Review with students the steps for solving a quadratic equation by completing the square. Remind students that when they check their answers algebraically, they should substitute the values for x in the original equation, not in the equation with the value added for a perfect-square trinomial. For irrational solutions, students can approximate the solutions and check their answers on a graphing calculator.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. Possible answer: Subtract c from both sides. Then add $\left(\frac{b}{2}\right)^2$ to both sides. Factor the perfect-square trinomial. Take the square root of both sides. Write two equations, using both the positive and negative square roots, and solve each equation.
2. See Additional Answers.

8-8 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. **Vocabulary** Describe in your own words how to *complete the square* for the equation $1 = x^2 + 4x$.

SEE EXAMPLE 1

Complete the square to form a perfect-square trinomial.

2. $x^2 + 14x +$ 49
3. $x^2 - 4x +$ 4
4. $x^2 - 3x +$ $\frac{9}{4}$

Solve by completing the square.

SEE EXAMPLE 2

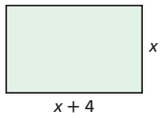
5. $x^2 + 6x = -5$ $-5, -1$
6. $x^2 - 8x = 9$ $-1, 9$
7. $x^2 + x = 30$ $-6, 5$
8. $x^2 + 2x = 21$ $-1 \pm \sqrt{22}$
9. $x^2 - 10x = -9$ $1, 9$
10. $x^2 + 16x = 91$ $-8 \pm \sqrt{155}$

SEE EXAMPLE 3

11. $-x^2 - 5x = -5$
12. $-x^2 - 3x + 2 = 0$
13. $-6x = 3x^2 + 9$ no real solutions
14. $2x^2 - 6x = -10$ no real solutions
15. $-x^2 + 8x - 6 = 0$ $4 \pm \sqrt{10}$
16. $4x^2 + 16 = -24x$ $-3 \pm \sqrt{5}$

SEE EXAMPLE 4

17. **Multi-Step** The length of a rectangle is 4 meters longer than the width. The area of the rectangle is 80 square meters. Find the length and width. Round your answers to the nearest tenth of a meter. 7.2 m; 11.2 m



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
18–20	1
21–26	2
27–32	3
33	4

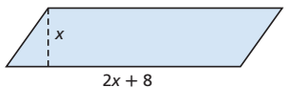
Complete the square to form a perfect-square trinomial.

18. $x^2 - 16x +$ 64
19. $x^2 - 2x +$ 1
20. $x^2 + 11x +$ $\frac{121}{4}$

Solve by completing the square.

21. $x^2 - 10x = 24$ $-2, 12$
22. $x^2 - 6x = -9$ 3
23. $x^2 + 15x = -26$ $-13, -2$
24. $x^2 + 6x = 16$ $-8, 2$
25. $x^2 - 2x = 48$ $-6, 8$
26. $x^2 + 12x = -36$ -6
27. $-x^2 + x + 6 = 0$ $-2, 3$
28. $2x^2 = -7x - 29$ no real solutions
29. $-x^2 - x + 1 = 0$
30. $3x^2 - 6x - 9 = 0$ $3, -1$
31. $-x^2 = 15x + 30$
32. $2x^2 + 20x - 10 = 0$

33. **Geometry** The base of a parallelogram is 8 inches longer than twice the height. The area of the parallelogram is 64 square inches. What is the height? 4 in.



Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Solve each equation by completing the square.

34. $3x^2 + x = 10$ $-2, \frac{5}{3}$
35. $x^2 = 2x + 6$ $1 \pm \sqrt{7}$
36. $2a^2 = 5a + 12$ $-\frac{3}{2}, 4$
37. $2x^2 + 5x = 3$ $-3, \frac{1}{2}$
38. $4x = 7 - x^2$ $-2 \pm \sqrt{11}$
39. $8x = -x^2 + 20$ $-10, 2$

40. **Hobbies** The height in feet h of a water bottle rocket launched from a rooftop is given by the equation $h = -16t^2 + 320t + 32$, where t is the time in seconds. After the rocket is fired, how long will it take to return to the ground? Solve by completing the square. Round your answer to the nearest tenth of a second. 20.1 s

Complete each trinomial so that it is a perfect square.

41. $x^2 + 18x +$ 81
42. $x^2 - 100x +$ 2500
43. $x^2 - 7x +$ $\frac{49}{4}$
44. $x^2 +$ 4 $x + 4$
45. $x^2 -$ 9 $x + \frac{81}{4}$
46. $x^2 +$ $\frac{1}{3}$ $x + \frac{1}{36}$

29. $x = \frac{-1 \pm \sqrt{5}}{2}$

31. $x = \frac{-15 \pm \sqrt{105}}{2}$

32. $x = \frac{-10 \pm \sqrt{120}}{2} = -5 \pm \sqrt{30}$

8-8 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–2
Basic 18–26, 41–43, 47
Average 18–26, 35, 41–47
Advanced 18–26, 35, 41–47, 60

If you finished Examples 1–4
Basic 18–43, 47–58, 61–66
Average 18–46, 48–67
Advanced 18–46, 48–72

Homework Quick Check
Quickly check key concepts.
Exercises: 18, 24, 28, 33, 48

Teaching Tip **Kinesthetic** For **Exercise 1**, students can use a model such as algebra tiles in their descriptions.

Answers

1. Divide 4 by 2 and square the result: $\left(\frac{4}{2}\right)^2 = 4$. Add 4 to both sides of the equation: $5 = x^2 + 4x + 4$.

11. $x = \frac{-5 \pm \sqrt{45}}{2} = \frac{-5 \pm 3\sqrt{5}}{2}$

12. $x = \frac{-3 \pm \sqrt{17}}{2}$

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 17, 33, 40, 47, 61, 72

Construct viable arguments and critique the reasoning of others. Exercises 49, 59–60, 62

Use appropriate tools strategically. Exercise 48

Look for and make use of structure. Exercise 70

Learn It Online
State Resources Online

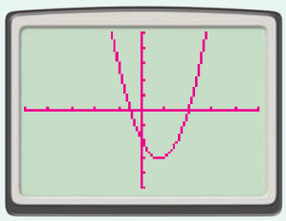
Teaching Tip **Kinesthetic** To determine the width and length of the combined area in **Exercise 47,** suggest that students run their finger horizontally across the diagram from left to right. The distance is x, then 10, then x again, so the expression for width is $x + 10 + x$, or $2x + 10$. Then have students repeat for the length.

Multi-Step Test Prep **Exercise 61** involves writing and solving a quadratic equation to solve a projectile problem. This exercise prepares students for the Multi-Step Test Prep.

Answers

48a. $-\frac{1}{2}$, 2

b.



c. Use the zero feature to find where the graph crosses the x-axis.

d. Possible answer: Completing the square takes several steps, but it produces the exact solutions. Creating the graph with a graphing calculator is fast, but you may get only approximate solutions.

47. Multi-Step A roped-off area of width x is created around a 34-by-10-foot rectangular museum display of Egyptian artifacts, as shown. The combined area of the display and the roped-off area is 640 square feet. $(10 + 2x)(34 + 2x) = 640$

a. Write an equation for the combined area.

b. Find the width of the roped-off area. 3 ft

48. Graphing Calculator Compare solving a quadratic equation by completing the square with finding the solutions on a graphing calculator.

a. Complete the square to solve $2x^2 - 3x - 2 = 0$.

b. Use your graphing calculator to graph $y = 2x^2 - 3x - 2$.

c. Explain how to use this graph to find the solutions of $2x^2 - 3x - 2 = 0$.

d. Compare the two methods of solving the equation. What are the advantages and disadvantages of each?

49. ERROR ANALYSIS Explain the error below. What is the correct answer?

$x^2 + 4x = 77$
$x^2 + 4x + 4 = 77 + 4$
$(x + 2)^2 = 81$
$x + 2 = 9$
$x = 7$

If $(x + 2)^2 = 81$, then $x + 2 = 9$ or $x + 2 = -9$. The correct answer is $x = 7$ or $x = -11$.

59. Possible answer: Completing the square to solve $x^2 + 11x + 18 = 0$ involves fractions, whereas completing the square to solve $x^2 + 20x - 21 = 0$ does not.

Solve each equation by completing the square. $-6 \pm \sqrt{26}$

50. $5x^2 - 50x = 55$ $-1, 11$

51. $3x^2 + 36x = -30$ $-4 \pm \sqrt{5}$

52. $28x - 2x^2 = 26$ 1, 13

53. $-36x = 3x^2 + 108$ -6

54. $0 = 4x^2 + 32x + 44$

55. $16x + 40 = -2x^2$ no real solutions

56. $x^2 + 5x + 6 = 10x$ 2, 3

57. $x^2 + 3x + 18 = -3x$ no real solutions

58. $4x^2 + x + 1 = 3x^2$ no real solutions

60. Add $2xy$. The perfect-square trinomial of $(x + y)(x + y)$ simplifies to $x^2 + xy + xy + y^2$, or $x^2 + 2xy + y^2$. This is the equivalent of $x^2 + y^2 + 2xy$.

59. Write About It Jamal prefers to solve $x^2 + 20x - 21 = 0$ by completing the square. Heather prefers to solve $x^2 + 11x + 18 = 0$ by factoring. Explain their reasoning.

60. Critical Thinking What should be done to the binomial $x^2 + y^2$ to make it a perfect-square trinomial? Explain.

Multi-Step Test Prep

61. The function $h(t) = -16t^2 + vt + c$ models the height in feet of a golf ball after t seconds when it is hit with initial vertical velocity v from initial height c feet. A golfer stands on a tee box that is 32 feet above the fairway. He hits the golf ball from the tee at an initial vertical velocity of 64 feet per second.

a. Write an equation that gives the time t when the golf ball lands on the fairway at height 0. $-16t^2 + 64t + 32 = 0$

b. What number would be added to both sides of the equation in part **a** to complete the square while solving for t? 4

c. Solve the equation from part **a** by completing the square to find the time it takes the ball to reach the fairway. Round to the nearest tenth of a second. 4.4 s

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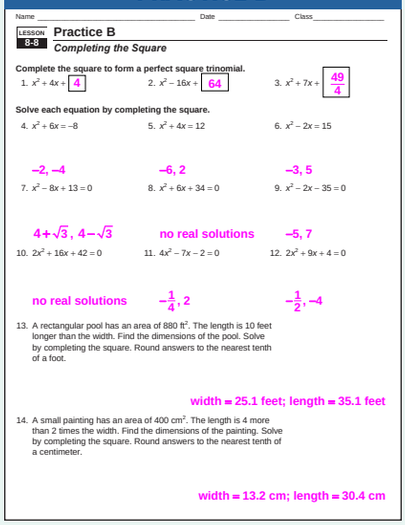
PRACTICE A

PRACTICE C

PRACTICE B



Name _____ Date _____ Class _____

LESSON 8-8 Practice B

Completing the Square

Complete the square to form a perfect square trinomial.

1. $x^2 + 4x +$ 4
2. $x^2 - 16x +$ 64
3. $x^2 + 7x +$ $\frac{49}{4}$

Solve each equation by completing the square.

4. $x^2 + 6x = -8$
5. $x^2 + 4x = 12$
6. $x^2 - 2x = 15$

–2, –4 | –6, 2 | –3, 5

7. $x^2 - 8x + 13 = 0$
8. $x^2 + 6x + 34 = 0$
9. $x^2 - 2x - 35 = 0$

$4 + \sqrt{3}$, $4 - \sqrt{3}$ | no real solutions | –5, 7

10. $2x^2 + 16x + 42 = 0$
11. $4x^2 - 7x - 2 = 0$
12. $2x^2 + 9x + 4 = 0$

no real solutions | $-\frac{1}{4}$, 2 | $-\frac{1}{2}$, –4

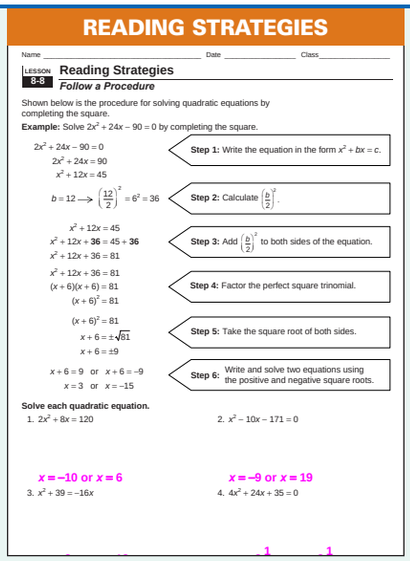
13. A rectangular pool has an area of 880 ft². The length is 10 feet longer than the width. Find the dimensions of the pool. Solve by completing the square. Round answers to the nearest tenth of a foot.

width ≈ 25.1 feet; length ≈ 35.1 feet

14. A small painting has an area of 400 cm². The length is 4 more than 2 times the width. Find the dimensions of the painting. Solve by completing the square. Round answers to the nearest tenth of a centimeter.

width ≈ 13.2 cm; length ≈ 30.4 cm

READING STRATEGIES



Name _____ Date _____ Class _____

LESSON 8-8 Reading Strategies

Follow a Procedure

Shown below is the procedure for solving quadratic equations by completing the square.

Example: Solve $2x^2 + 24x - 90 = 0$ by completing the square.

$2x^2 + 24x - 90 = 0$

$2x^2 + 24x = 90$

$x^2 + 12x = 45$

$b = 12 \rightarrow \left(\frac{12}{2}\right)^2 = 6^2 = 36$

Step 1: Write the equation in the form $x^2 + bx = c$.

Step 2: Calculate $\left(\frac{b}{2}\right)^2$.

$x^2 + 12x = 45$

$x^2 + 12x + \mathbf{36} = 45 + \mathbf{36}$

$x^2 + 12x + 36 = 81$

Step 3: Add $\left(\frac{b}{2}\right)^2$ to both sides of the equation.

$x^2 + 12x + 36 = 81$

$(x + 6)(x + 6) = 81$

$(x + 6)^2 = 81$

Step 4: Factor the perfect square trinomial.

$(x + 6)^2 = 81$

$x + 6 = \pm\sqrt{81}$

$x + 6 = \pm 9$

Step 5: Take the square root of both sides.

$x + 6 = 9$ or $x + 6 = -9$

$x = 3$ or $x = -15$

Step 6: Write and solve two equations using the positive and negative square roots.

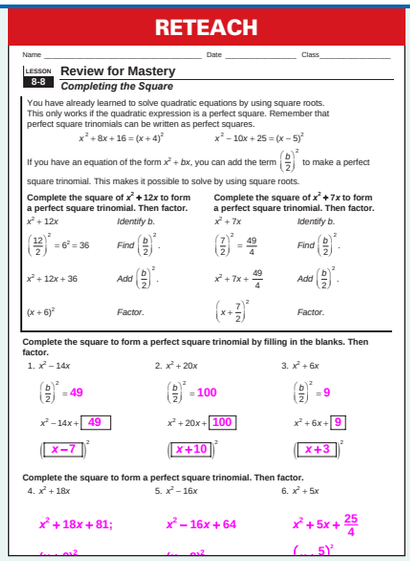
Solve each quadratic equation.

1. $2x^2 + 8x = 120$
2. $x^2 - 10x - 171 = 0$

$x = -10$ or $x = 6$ | $x = -9$ or $x = 19$

3. $x^2 + 39 = -16x$
4. $4x^2 + 24x + 35 = 0$

RETEACH



Name _____ Date _____ Class _____

LESSON 8-8 Review for Mastery

Completing the Square

You have already learned to solve quadratic equations by using square roots. This only works if the quadratic expression is a perfect square. Remember that perfect square trinomials can be written as perfect squares.

$x^2 + 8x + 16 = (x + 4)^2$ $x^2 - 10x + 25 = (x - 5)^2$

If you have an equation of the form $x^2 + bx$, you can add the term $\left(\frac{b}{2}\right)^2$ to make a perfect square trinomial. This makes it possible to solve by using square roots.

Complete the square of $x^2 + 12x$ to form a perfect square trinomial. Then factor.

$x^2 + 12x$	*Identify b.*
$\left(\frac{12}{2}\right)^2 = 6^2 = 36$	*Find* $\left(\frac{b}{2}\right)^2$.
$x^2 + 12x + 36$	*Add* $\left(\frac{b}{2}\right)^2$.
$(x + 6)^2$	*Factor.*

Complete the square of $x^2 + 7x$ to form a perfect square trinomial. Then factor.

$x^2 + 7x$	*Identify b.*
$\left(\frac{7}{2}\right)^2 = \frac{49}{4}$	*Find* $\left(\frac{b}{2}\right)^2$.
$x^2 + 7x + \frac{49}{4}$	*Add* $\left(\frac{b}{2}\right)^2$.
$\left(x + \frac{7}{2}\right)^2$	*Factor.*

Complete the square to form a perfect square trinomial by filling in the blanks. Then factor.

1. $x^2 - 14x$
   $\left(\frac{b}{2}\right)^2 =$ 49
   $x^2 - 14x +$ 49
   $(x -$ 7$)^2$
2. $x^2 + 20x$
   $\left(\frac{b}{2}\right)^2 =$ 100
   $x^2 + 20x +$ 100
   $(x +$ 10$)^2$
3. $x^2 + 6x$
   $\left(\frac{b}{2}\right)^2 =$ 9
   $x^2 + 6x +$ 9
   $(x +$ 3$)^2$

Complete the square to form a perfect square trinomial. Then factor.

4. $x^2 + 18x$ — $x^2 + 18x + 81$;
5. $x^2 - 16x$ — $x^2 - 16x + 64$
6. $x^2 + 5x$ — $x^2 + 5x + \frac{25}{4}$

62. Write About It Compare solving an equation of the form $x^2 + bx + c = 0$ by completing the square and solving an equation of the form $ax^2 + bx + c = 0$ by completing the square. **When solving an equation of the form $ax^2 + bx + c = 0$, you must first divide the equation by the coefficient a.**

TEST PREP

63. What value of c will make $x^2 + 16x + c$ a perfect-square trinomial?

Ⓐ 32 Ⓑ 64 Ⓒ 128 Ⓓ 256

64. What value of b will make $x^2 + b + 25$ a perfect-square trinomial?

Ⓕ 5 Ⓖ $5x$ Ⓗ 10 Ⓙ $10x$

65. Which of the following is closest to a solution of $3x^2 + 2x - 4 = 0$?

Ⓐ 0 Ⓑ 1 Ⓒ 2 Ⓓ 3

66. Short Response Solve $x^2 - 8x - 20 = 0$ by completing the square. Explain each step in your solution. **$-2, 10$**

CHALLENGE AND EXTEND

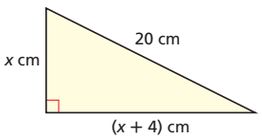
Solve each equation by completing the square.

67. $6x^2 + 5x = 6$ $-\frac{3}{2}, \frac{2}{3}$ **68.** $7x + 3 = 6x^2$ $-\frac{1}{3}, \frac{3}{2}$ **69.** $4x = 1 - 3x^2$ $-\frac{2}{3} \pm \frac{\sqrt{7}}{3}$

70. What should be done to the binomial $ax^2 + bx$ to obtain a perfect-square trinomial?

71. Solve $ax^2 + bx = 0$ for x. **$0, -\frac{b}{a}$**

72. Geometry The hypotenuse of a right triangle is 20 cm. One of the legs is 4 cm longer than the other leg. Find the area of the triangle. (*Hint:* Use the Pythagorean Theorem.) **96 cm²**



TEST PREP DOCTOR In **Exercise 64,** explain to students that in this equation, b stands for the x-term, not the coefficient of x.

Explain that the correct answer to **Exercise 65** is the value of x that makes the left side of the equation closest to zero.

Answers

70. Divide both terms by a:
$x^2 + \frac{b}{a}x$.
Then add half of $\frac{b}{a}$ squared:
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2$.

Journal

Have students explain what it means to complete the square.

ALTERNATIVE ASSESSMENT

Have students write an equation of the form $x^2 + bx = c$ that cannot be solved by factoring. Have them solve the equation by completing the square and justify each step.

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8-8 Lesson Quiz

Complete the square to form a perfect-square trinomial.

1. $x^2 + 11x +$ ▢ $\frac{121}{4}$

2. $x^2 - 18x +$ ▢ 81

Solve by completing the square.

3. $x^2 - 2x - 1 = 0$
$\frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$

4. $3x^2 + 6x = 144$ $6, -8$

5. $4x^2 + 44x = 23$ $\frac{1}{2}, -\frac{23}{2}$

6. Dymond is painting a rectangular banner for a football game. She has enough paint to cover 120 ft². She wants the length of the banner to be 7 ft longer than the width. What dimensions should Dymond use for the banner?
8 ft by 15 ft

Also available online

PROBLEM SOLVING

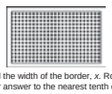
Name _____ Date _____ Class _____

LESSON 8-8 **Problem Solving**
Completing the Square

The Ward family is redecorating several rooms of their house. Write the correct answer.

1. The Wards decided to use carpet tiles in the family room. The room has an area of 176 square feet and is 5 feet longer than it is wide. Find the dimensions of the family room.
11 ft by 16 ft

2. Angelique wants to have a rug that is 9 feet long and 7 feet wide in her bedroom. The rug will cover the whole floor except a border that is x feet wide. The area of her room is 167 square feet.



Find the width of the border, x. Round your answer to the nearest tenth of a foot.
2.5 ft

3. Giselle is going to frame a portrait of the family and place it on the mantle in the family room. The portrait is 10 inches longer than it is tall and will take up a total area of 1344 square inches once it is inside the 2 inch thick frame. Find the dimensions and area of the unframed portrait.
38 in. by 28 in.
1064 square in.

Select the best answer.

4. The landing for the steps leading up to a county courthouse is shaped like a trapezoid. The area of the landing is 1500 square feet. The shorter base of the trapezoid is 15 feet longer than the height. The longer base is 5 feet longer than 3 times the height. What is the length of the longer base?
A 25 feet C 80 feet
B 40 feet D 95 feet

5. The height of a pumpkin launched from a cannon is given by the function $h = -16t^2 + 240t + 16$ where t is the time in seconds. How many seconds is the pumpkin in the air? Round your answer to the nearest tenth of a second.
F 7.5 seconds H 16 seconds
G 15.1 seconds J 32 seconds

6. Georgia works part-time at a daycare while she is going to college. She earned $160 last week. Georgia worked 12 more hours than the amount she is paid per hour. What is Georgia's hourly pay rate?
A $6.00 C $12.00
B $8.00 D $20.00

7. Part of the set for a play is a triangular piece of plywood. The area of the triangle is 20 square feet. The base is 3 feet longer than the height. What is the height of the triangle? Round your answer to the nearest tenth of a foot.
F 3 feet H 3.9 feet
G 3.2 feet J 5 feet

CHALLENGE

Name _____ Date _____ Class _____

LESSON 8-8 **Challenge**
Using Coordinates of the Vertex to Explore X-intercepts

To write $y = x^2 - 7x + 3$ in the form $y = (x - h)^2 + k$, you could follow these steps.

$y = x^2 - 7x + 3$

$y = x^2 - 7x + \left(\frac{-7}{2}\right)^2 + 3 - \left(\frac{-7}{2}\right)^2$

$y = \left(x - \frac{7}{2}\right)^2 + 3 - \frac{7^2}{4}$

Notice that the number outside the parentheses has not been simplified; leaving it in this form will help you to see a pattern in the following exercises.

Use the method shown above to rewrite each function in the form $y = (x - h)^2 + k$.

1. $y = x^2 + 5x - 1$
$y = \left(x + \frac{5}{2}\right)^2 - 1$
$-\frac{5^2}{4}$

2. $y = x^2 - 11x + 6$
$y = \left(x - \frac{11}{2}\right)^2 + 6$
$-\frac{11^2}{4}$

3. $y = x^2 + bx + c$
$y = \left(x + \frac{b}{2}\right)^2 + c$
$-\frac{b^2}{4}$

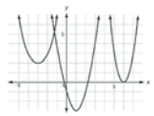
4. Recall that the vertex of the graph of $y = (x - h)^2 + k$ is at (h, k). Use your answer to Exercise 3 to give the coordinates of the vertex of the graph of $y = x^2 + bx + c$ in terms of b and c. $\left(\frac{-b}{2}, c - \frac{b^2}{4}\right)$

For Exercises 5–7, consider the parabola whose equation is $y = ax^2 + bx + c$.

5. How do you know that the graph of this parabola opens upward? $a > 0$

6. How does the y-coordinate of the vertex show whether the parabola crosses the x-axis?
If the parabola opens upward and the y-coordinate of the vertex is greater than 0, then the parabola is completely above the x-axis. If it is less than 0, then the parabola crosses the x-axis at two points. If it is equal to 0, then the parabola touches the x-axis at one point.

7. How many x-intercepts does the graph of $y = x^2 + bx + c$ have if you know that
a. $c - \frac{b^2}{4} > 0$? 0
b. $c - \frac{b^2}{4} = 0$? 1
c. $c - \frac{b^2}{4} < 0$? 2



Use your answers to Exercises 4 and 7 to determine (a) the coordinates of the vertex and (b) the number of x-intercepts for each of the following functions.

8-9 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Solve quadratic equations by using the Quadratic Formula.

Determine the number of solutions of a quadratic equation by using the discriminant.

Online Edition
Tutorial Videos, Interactivity

Countdown

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Warm Up

Evaluate for $x = -2$, $y = 3$, and $z = -1$.

1. x^2 4
2. xyz 6
3. $x^2 - yz$ 7
4. $y - xz$ 1
5. $-x$ 2
6. $z^2 - xy$ 7

Also available online

Math Humor

Q: What do you call a person who is crazy about solving quadratic equations?

A: A quadratic fanatic.

COMMON CORE

CC.9-12.A.REI.4 Solve quadratic equations in one variable.
a. Use the method of completing the square …. Derive the quadratic formula from this form.
b. Solve quadratic equations by inspection …, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation….

CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution….

Learn It Online
State Resources Online

COMMON CORE

8-9

The Quadratic Formula and the Discriminant

CC.9-12.A.REI.4b Solve quadratic equations by … the quadratic formula…. *Also* **CC.9-12.A.REI.4a, CC.9-12.A.REI.1**

Objectives
Solve quadratic equations by using the Quadratic Formula.

Determine the number of solutions of a quadratic equation by using the discriminant.

Vocabulary
discriminant

Why learn this?
You can use the discriminant to determine whether the weight in a carnival strength test will reach a certain height. (See Exercise 4.)

Previously, you completed the square to solve quadratic equations. If you complete the square of $ax^2 + bx + c = 0$, you can derive the *Quadratic Formula*. The Quadratic Formula can be used to solve *any* quadratic equation.

Numbers		Algebra
$2x^2 + 6x + 1 = 0$		$ax^2 + bx + c = 0,\ a \neq 0$
$\frac{2}{2}x^2 + \frac{6}{2}x + \frac{1}{2} = \frac{0}{2}$	*Divide both sides by a.*	$\frac{a}{a}x^2 + \frac{b}{a}x + \frac{c}{a} = \frac{0}{a}$
$x^2 + 3x + \frac{1}{2} = 0$		$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
$x^2 + 3x = -\frac{1}{2}$	*Subtract $\frac{c}{a}$ from both sides.*	$x^2 + \frac{b}{a}x = -\frac{c}{a}$
$x^2 + 3x + \left(\frac{3}{2}\right)^2 = -\frac{1}{2} + \left(\frac{3}{2}\right)^2$	*Complete the square.*	$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
$\left(x + \frac{3}{2}\right)^2 = \frac{9}{4} - \frac{1}{2}$	*Factor and simplify.*	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$
$\left(x + \frac{3}{2}\right)^2 = \frac{9}{4} - \frac{2}{4}$	*Use common denominators.*	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$
$\left(x + \frac{3}{2}\right)^2 = \frac{7}{4}$	*Simplify.*	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$
$x + \frac{3}{2} = \pm\frac{\sqrt{7}}{2}$	*Take square roots.*	$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$
$x = -\frac{3}{2} \pm \frac{\sqrt{7}}{2}$	*Subtract $\frac{b}{2a}$ from both sides.*	$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-3 \pm \sqrt{7}}{2}$	*Simplify.*	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Remember!
To add fractions, you need a common denominator.

$$\frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2}{4a^2} - \frac{c}{a}\left(\frac{4a}{4a}\right) = \frac{b^2}{4a^2} - \frac{4ac}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

Know it! Note

The Quadratic Formula

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

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1 Introduce

EXPLORATION

The Quadratic Formula and the Discriminant

Use a graphing calculator to explore quadratic equations of the form $ax^2 + bx + c = 0$.

1. Graph the related function for each of the quadratic equations in the table to determine how many real solutions each equation has. Then complete the table as shown.

Quadratic Equation	Number of Real Solutions	a	b	c	$b^2 - 4ac$	Is $b^2 - 4ac$ positive, negative, or zero?
$x^2 + 2x - 8 = 0$	2	1	2	−8	36	Positive
$x^2 - 8x + 16 = 0$	1	1	−8	16	0	Zero
$x^2 + 6x + 11 = 0$	0	1	6	11	−8	Negative
$2x^2 + 3x + 3 = 0$	0	2	3	3	−15	Negative
$4x^2 - 8x + 3 = 0$	2	4	−8	3	16	Positive
$4x^2 + 4x + 1 = 0$	1	4	4	1	0	Zero

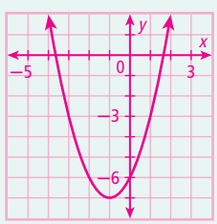
2. Look for a relationship between the number of real solutions and the value of $b^2 - 4ac$. What do you notice?

THINK AND DISCUSS

3. **Discuss** what you can conclude about a quadratic equation for which $b^2 - 4ac > 0$. The equation has 2 real solutions.
4. **Explain** what you know about the value of $b^2 - 4ac$ for a

Motivate

Have students graph $y = x^2 + 2x - 6$.



Show students that the solutions of the related quadratic equation are not integers. Tell students they can find exact solutions to any quadratic equation by using the Quadratic Formula.

Explorations and answers are provided in your online resources.

EXAMPLE 1 Using the Quadratic Formula

Solve using the Quadratic Formula.

A $2x^2 + 3x - 5 = 0$

$2x^2 + 3x + (-5) = 0$ — *Identify a, b, and c.*

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — *Use the Quadratic Formula.*

$x = \dfrac{-3 \pm \sqrt{3^2 - 4(2)(-5)}}{2(2)}$ — *Substitute 2 for a, 3 for b, and −5 for c.*

$x = \dfrac{-3 \pm \sqrt{9 - (-40)}}{4}$ — *Simplify.*

$x = \dfrac{-3 \pm \sqrt{49}}{4} = \dfrac{-3 \pm 7}{4}$ — *Simplify.*

$x = \dfrac{-3 + 7}{4}$ or $x = \dfrac{-3 - 7}{4}$ — *Write as two equations.*

$x = 1$ or $x = -\dfrac{5}{2}$ — *Simplify.*

Helpful Hint

You can graph the related quadratic function to see if your solutions are reasonable.

B $2x = x^2 - 3$

$1x^2 + (-2)x + (-3) = 0$ — *Write in standard form. Identify a, b, and c.*

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}$ — *Substitute 1 for a, −2 for b, and −3 for c.*

$x = \dfrac{2 \pm \sqrt{4 - (-12)}}{2}$ — *Simplify.*

$x = \dfrac{2 \pm \sqrt{16}}{2} = \dfrac{2 \pm 4}{2}$ — *Simplify.*

$x = \dfrac{2 + 4}{2}$ or $x = \dfrac{2 - 4}{2}$ — *Write as two equations.*

$x = 3$ or $x = -1$ — *Simplify.*

CHECK IT OUT! **Solve using the Quadratic Formula.**

1a. $-3x^2 + 5x + 2 = 0$ $2, -\frac{1}{3}$ **1b.** $2 - 5x^2 = -9x$ $2, -\frac{1}{5}$

Many quadratic equations can be solved by graphing, factoring, taking the square root, or completing the square. Some cannot be easily solved by any of these methods, but you can use the Quadratic Formula to solve any quadratic equation.

EXAMPLE 2 Using the Quadratic Formula to Estimate Solutions

Solve $x^2 - 2x - 4 = 0$ using the Quadratic Formula.

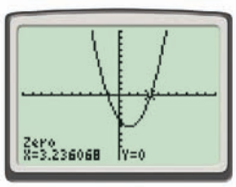
$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{4 - (-16)}}{2} = \dfrac{2 \pm \sqrt{20}}{2}$

$x = \dfrac{2 + \sqrt{20}}{2}$ or $x = \dfrac{2 - \sqrt{20}}{2}$

Use a calculator: $x \approx 3.24$ or $x \approx -1.24$.

Check reasonableness



2. $\dfrac{8 - \sqrt{56}}{4} \approx 0.13$, $\dfrac{8 + \sqrt{56}}{4} \approx 3.87$

CHECK IT OUT! **2.** Solve $2x^2 - 8x + 1 = 0$ using the Quadratic Formula.

COMMON ERROR ALERT

Students might make sign errors in finding $-b$ when b is negative. Encourage students to use parentheses and to write down every step.

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Additional Examples

Example 1

Solve using the Quadratic Formula.

A. $6x^2 + 5x - 4 = 0$ $-\frac{4}{3}, \frac{1}{2}$

B. $x^2 = x + 20$ $-4, 5$

Example 2

Solve $x^2 + 3x - 7 = 0$ by using the Quadratic Formula.

$\dfrac{-3 + \sqrt{37}}{2} \approx 1.54$,

$\dfrac{-3 - \sqrt{37}}{2} \approx -4.54$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLES 1–2

- How can you check your answer?
- Why is the symbol $\pm$ in the Quadratic Formula?
- Where does the Quadratic Formula come from?

Teaching Tip **Communicating Math** Make sure that students know what an exact answer is. In **Example 2,** $\frac{2 + \sqrt{20}}{2}$ is an exact solution; 3.24 is an approximation.

2 Teach

Guided Instruction

Let students know that any quadratic equation can be solved by using the Quadratic Formula and that it will always give exact solutions, but it may not always be the quickest method. Caution students to pay attention to signs when using the Quadratic Formula to avoid simple mistakes. Point out the benefit of finding the discriminant first so that students will be aware of the number of real solutions.

Reaching All Learners

Through Auditory Cues

Help students remember the Quadratic Formula by singing it to the tune of "Row, Row, Row Your Boat" or "Pop Goes the Weasel."

x equals negative *b*
plus or minus square root
of *b* squared minus four *a c*
all over two *a*.

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Additional Examples

Example 3

Find the number of real solutions of each equation using the discriminant.

A. $3x^2 - 2x + 2 = 0$ 0

B. $2x^2 + 11x + 12 = 0$ 2

C. $x^2 + 8x + 16 = 0$ 1

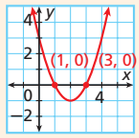
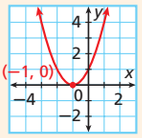
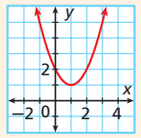
Also available online

Questioning Strategies

EXAMPLE 3

- What part of the Quadratic Formula does the discriminant come from?
- Explain why the value of the discriminant indicates the number of solutions.

If the quadratic equation is in standard form, the **discriminant** of a quadratic equation is $b^2 - 4ac$, the part of the equation under the radical sign. Recall that quadratic equations can have two, one, or no real solutions. You can determine the number of solutions of a quadratic equation by evaluating its discriminant.

Equation	$x^2 - 4x + 3 = 0$	$x^2 + 2x + 1 = 0$	$x^2 - 2x + 2 = 0$
Discriminant	$a = 1, b = -4, c = 3$ $b^2 - 4ac$ $(-4)^2 - 4(1)(3)$ $16 - 12$ 4 The discriminant is positive.	$a = 1, b = 2, c = 1$ $b^2 - 4ac$ $2^2 - 4(1)(1)$ $4 - 4$ 0 The discriminant is zero.	$a = 1, b = -2, c = 2$ $b^2 - 4ac$ $(-2)^2 - 4(1)(2)$ $4 - 8$ -4 The discriminant is negative.
Graph of Related Function	Notice that the related function has two x-intercepts. 	Notice that the related function has one x-intercept. 	Notice that the related function has no x-intercepts. 
Number of Solutions	two real solutions	one real solution	no real solutions

Know it! Note

The Discriminant of Quadratic Equation $ax^2 + bx + c = 0$

If $b^2 - 4ac > 0$, the equation has two real solutions.
If $b^2 - 4ac = 0$, the equation has one real solution.
If $b^2 - 4ac < 0$, the equation has no real solutions.

EXAMPLE 3 Using the Discriminant

Find the number of real solutions of each equation using the discriminant.

A $3x^2 + 10x + 2 = 0$
$a = 3, b = 10, c = 2$
$b^2 - 4ac$
$10^2 - 4(3)(2)$
$100 - 24$
76
$b^2 - 4ac$ is positive.
There are two real solutions.

B $9x^2 - 6x + 1 = 0$
$a = 9, b = -6, c = 1$
$b^2 - 4ac$
$(-6)^2 - 4(9)(1)$
$36 - 36$
0
$b^2 - 4ac$ is zero.
There is one real solution.

C $x^2 + x + 1 = 0$
$a = 1, b = 1, c = 1$
$b^2 - 4ac$
$1^2 - 4(1)(1)$
$1 - 4$
-3
$b^2 - 4ac$ is negative.
There are no real solutions.

CHECK IT OUT!

Find the number of real solutions of each equation using the discriminant.

3a. $2x^2 - 2x + 3 = 0$ 0 **3b.** $x^2 + 4x + 4 = 0$ 1 **3c.** $x^2 - 9x + 4 = 0$ 2

Reaching All Learners

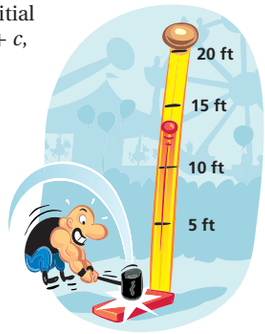
Through Communication

Discuss with students why each type of discriminant indicates the number of real solutions that it does.

If the discriminant is negative, you cannot take its square root, so there are no real solutions.

Have students give explanations for the number of real solutions when the discriminant is positive or 0.

The height h in feet of an object shot straight up with initial velocity v in feet per second is given by $h = -16t^2 + vt + c$, where c is the beginning height of the object above the ground.



EXAMPLE 4 Physics Application

A weight 1 foot above the ground on a carnival strength test is shot straight up with an initial velocity of 35 feet per second. Will it ring the bell at the top of the pole? Use the discriminant to explain your answer.

Helpful Hint

If the object is shot straight up from the ground, the initial height of the object above the ground equals 0.

$h = -16t^2 + vt + c$

$20 = -16t^2 + 35t + 1$ *Substitute 20 for h, 35 for v, and 1 for c.*

$0 = -16t^2 + 35t + (-19)$ *Subtract 20 from both sides.*

$b^2 - 4ac$ *Evaluate the discriminant.*

$35^2 - 4(-16)(-19) = 9$ *Substitute −16 for a, 35 for b, and −19 for c.*

The discriminant is positive, so the equation has two solutions. The weight will reach a height of 20 feet so it will ring the bell.

CHECK IT OUT! **4. What if...?** Suppose the weight is shot straight up with an initial velocity of 20 feet per second. Will it ring the bell? Use the discriminant to explain your answer.

No; the discriminant is negative so the weight will not ring the bell.

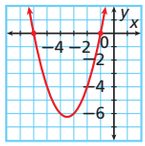
There is no one correct way to solve a quadratic equation. Many quadratic equations can be solved using several different methods.

EXAMPLE 5 Solving Using Different Methods

Solve $x^2 + 7x + 6 = 0$.

Method 1 Solve by graphing.

$y = x^2 + 7x + 6$ *Write the related quadratic function and graph it.*



The solutions are the x-intercepts, -6 and -1.

Method 2 Solve by factoring.

$x^2 + 7x + 6 = 0$

$(x + 6)(x + 1) = 0$ *Factor.*

$x + 6 = 0$ or $x - 1 = 0$ *Use the Zero Product Property.*

$x = -6$ or $x = -1$ *Solve each equation.*

Method 3 Solve by completing the square.

$x^2 + 7x + 6 = 0$

$x^2 + 7x = -6$

$x^2 + 7x + \frac{49}{4} = -6 + \frac{49}{4}$ *Add $\left(\frac{b}{2}\right)^2$ to both sides.*

$\left(x + \frac{7}{2}\right)^2 = \frac{25}{4}$ *Factor and simplify.*

$x + \frac{7}{2} = \pm\frac{5}{2}$ *Take the square root of both sides.*

$x + \frac{7}{2} = \frac{5}{2}$ or $x + \frac{7}{2} = -\frac{5}{2}$ *Solve each equation.*

$x = -1$ or $x = -6$

COMMON ERROR ALERT

When solving problems like **Example 4,** students may forget to write the equation in standard form after they substitute the value for height. Remind them that the quadratic equation must be in the form $ax^2 + bx + c = 0$ before they can begin to use the Quadratic Formula.

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Additional Examples

Example 4

The height h in feet of an object shot straight up with initial velocity v in feet per second is given by $h = -16t^2 + vt + c$, where c is the initial height of the object above the ground. The ringer on a carnival strength test is 2 feet off the ground and is shot upward with an initial velocity of 30 feet per second. Will it reach a height of 20 feet? Use the discriminant to explain your answer.

The discriminant of $20 = -16t^2 + 30t + 2$ is negative, so there are no real solutions. The ringer will not reach a height of 20 feet.

Example 5

Solve $x^2 - 9x + 20 = 0$. 4, 5

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 4

- What would a positive discriminant indicate in this situation?
- What would a negative discriminant indicate?

EXAMPLE 5

- Which of the methods for solving a quadratic equation give an exact answer every time? Explain.
- What are the advantages and disadvantages of each of the methods?
- Which method do you like best? Why?

Teaching Tip **Inclusion** To help prevent computational errors, some students find it helpful to circle everything after b^2, calculate that, and then add that answer to b^2.

For example,

$\sqrt{7^2 - 4(2)(-3)}$

$= \sqrt{49 + 24}$

$= \sqrt{73}$

Method 4 Solve using the Quadratic Formula.

$1x^2 + 7x + 6 = 0$ *Identify a, b, and c.*

$x = \dfrac{-7 \pm \sqrt{7^2 - 4(1)(6)}}{2(1)}$ *Substitute 1 for a, 7 for b, and 6 for c.*

$x = \dfrac{-7 \pm \sqrt{49 - 24}}{2} = \dfrac{-7 \pm \sqrt{25}}{2} = \dfrac{-7 \pm 5}{2}$ *Simplify.*

$x = \dfrac{-7 + 5}{2}$ or $x = \dfrac{-7 - 5}{2}$ *Write as two equations.*

$x = -1$ or $x = -6$ *Solve each equation.*

CHECK IT OUT! **Solve.**

5a. $x^2 + 7x + 10 = 0$ $-2, -5$ **5b.** $-14 + x^2 = 5x$ $-2, 7$ **5c.** $2x^2 + 4x - 21 = 0$

5c. $\dfrac{-4 - \sqrt{184}}{4} \approx -4.39$, $\dfrac{-4 + \sqrt{184}}{4} \approx 2.39$

Notice that all of the methods in Example 5 produce the same solutions, -1 and -6. The only method you cannot use to solve $x^2 + 7x + 6 = 0$ is using square roots. Sometimes one method is better for solving certain types of equations. The table below gives some advantages and disadvantages of the different methods.

Methods of Solving Quadratic Equations

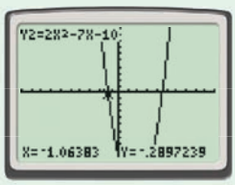
METHOD	ADVANTAGES	DISADVANTAGES
Graphing	• Always works to give approximate solutions • Can quickly see the number of solutions	• Cannot always get an exact solution
Factoring	• Good method to try first • Straightforward if the equation is factorable	• Complicated if the equation is not easily factorable • Not all quadratic equations are factorable.
Using square roots	• Quick when the equation has no x-term	• Cannot easily use when there is an x-term
Completing the square	• Always works	• Sometimes involves difficult calculations
Using the Quadratic Formula	• Always works • Can always find exact solutions	• Other methods may be easier or less time consuming.

Student to Student *Solving Quadratic Equations*

Binh Pham
Johnson High School

No matter what method I use, I like to check my answers for reasonableness by graphing.

I used the Quadratic Formula to solve $2x^2 - 7x - 10 = 0$. I found that $x \approx -1.09$ and $x \approx 4.59$. Then I graphed $y = 2x^2 - 7x - 10$. The x-intercepts appeared to be close to -1 and 4.5, so I knew my solutions were reasonable.



Comstock Images/Getty Images

3 Close

Summarize

Ask students to give a, b, and c for the following quadratic equations. Then use the discriminant to find the number of real solutions.

$x^2 + 3x = -7$ $a = 1, b = 3, c = 7$; no real solutions

$3x^2 = 5x + 10$ $a = 3, b = -5, c = -10$; two real solutions

$12x + 2 = -18x^2$ $a = 18, b = 12, c = 2$; one real solution

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

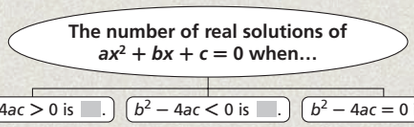
THINK AND DISCUSS

MATHEMATICAL PRACTICES

1. Describe how to use the discriminant to find the number of real solutions to a quadratic equation.
2. Choose a method to solve $x^2 + 5x + 4 = 0$ and explain why you chose that method.
3. Describe how the discriminant can be used to determine if an object will reach a given height.

Know it! Note

4. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write the number of real solutions.



8-9 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. **Vocabulary** If the *discriminant* is negative, the quadratic equation has _____?_____ real solution(s). (*no*, *one*, or *two*) no

Solve using the Quadratic Formula.

SEE EXAMPLE 1

2. $x^2 - 5x + 4 = 0$ 1, 4
3. $2x^2 = 7x - 3$ $\frac{1}{2}$, 3
4. $x^2 - 6x - 7 = 0$ −1, 7
5. $x^2 = -14x - 40$ −4, −10
6. $3x^2 - 2x = 8$ $-\frac{4}{3}$, 2
7. $4x^2 - 4x - 3 = 0$ $-\frac{1}{2}$, $\frac{3}{2}$

SEE EXAMPLE 2

8. $2x^2 - 6 = 0$
9. $x^2 + 6x + 3 = 0$
10. $x^2 - 7x + 2 = 0$
11. $3x^2 = -x + 5$
12. $x^2 - 4x - 7 = 0$
13. $2x^2 + x - 5 = 0$

SEE EXAMPLE 3

Find the number of real solutions of each equation using the discriminant.

14. $2x^2 + 4x + 3 = 0$ 0
15. $x^2 + 4x + 4 = 0$ 1
16. $2x^2 - 11x + 6 = 0$ 2
17. $x^2 + x + 1 = 0$ 0
18. $3x^2 = 5x - 1$ 2
19. $-2x + 3 = 2x^2$ 2
20. $2x^2 + 12x = -18$ 1
21. $5x^2 + 3x = -4$ 0
22. $8x = 1 - x^2$ 2

SEE EXAMPLE 4

23. **Hobbies** The height above the ground in meters of a model rocket on a particular launch can be modeled by the equation $h = -4.9t^2 + 102t + 100$, where t is the time in seconds after its engine burns out 100 m above the ground. Will the rocket reach a height of 600 m? Use the discriminant to explain your answer. Yes; the discriminant of $600 = -4.9t^2 + 102t + 100$ is positive so the rocket will reach the given height.

SEE EXAMPLE 5

Solve.

24. $x^2 + x - 12 = 0$ −4, 3
25. $x^2 + 6x + 9 = 0$ −3
26. $2x^2 - x - 1 = 0$ 1, $-\frac{1}{2}$
27. $4x^2 + 4x + 1 = 0$ $-\frac{1}{2}$
28. $2x^2 + x - 7 = 0$
29. $9 = 2x^2 + 3x$ −3, $\frac{3}{2}$

Answers

8. $x = \pm\sqrt{3} \approx \pm 1.73$
9. $x = \frac{-6 - \sqrt{24}}{2} \approx -5.45$, $x = \frac{-6 + \sqrt{24}}{2} \approx -0.55$
10. $x = \frac{7 - \sqrt{41}}{2} \approx 0.30$, $x = \frac{7 + \sqrt{41}}{2} \approx 6.70$
11. $x = \frac{-1 - \sqrt{61}}{6} \approx -1.47$, $x = \frac{-1 + \sqrt{61}}{6} \approx -1.14$
12. $x = \frac{4 - \sqrt{44}}{2} \approx -1.32$, $x = \frac{4 + \sqrt{44}}{2} \approx 5.32$
13. $x = \frac{-1 - \sqrt{41}}{4} \approx -1.85$, $x = \frac{-1 + \sqrt{41}}{4} \approx 1.35$
28. $x = \frac{-1 - \sqrt{57}}{4} \approx -2.14$, $x = \frac{-1 + \sqrt{57}}{4} \approx 1.64$

Answers to *Think and Discuss*

Possible answers:

1. If $b^2 - 4ac > 0$, there are 2 real solutions. If $b^2 - 4ac = 0$, there is 1 real solution. If $b^2 - 4ac < 0$, there are no real solutions.
2. $x = -1$ or $x = -4$; factoring method; least number of steps
3. If the discriminant of the equation describing the object's height is negative, there are no real solutions and the object will not reach the given height. If the discriminant is 0 or positive, there are 1 or 2 solutions and the object will reach the given height.
4. See Additional Answers.

8-9 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–3
Basic 30–38, 43–52, 56
Average 30–38, 43–52, 56
Advanced 30–38, 43–52, 55

If you finished Examples 1–5
Basic 30–54, 56–59
Average 30–59
Advanced 30–51, 53–55, 57–61

Homework Quick Check
Quickly check key concepts.
Exercises: 30, 34, 38, 39, 42, 53

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 23, 39, 53, 56, 60–61

Construct viable arguments and critique the reasoning of others. Exercises 54–55

Learn It Online
State Resources Online

MULTI-STEP TEST PREP **Exercise 56** involves evaluating a discriminant to determine whether a given height is possible. This exercise prepares students for the Multi-Step Test Prep.

Answers

33. $x = \frac{4 - \sqrt{12}}{2} \approx 0.27$, $x = \frac{4 + \sqrt{12}}{2} \approx 3.73$

34. $x = \pm\frac{\sqrt{60}}{6} \approx \pm 1.29$

35. $x = \frac{-7 - \sqrt{17}}{4} \approx -2.78$, $x = \frac{-7 + \sqrt{17}}{4} \approx -0.72$

54. 2; 0; 1; $x^2 = k$ in standard form is $1x^2 + 0x - k = 0$, so $a = 1$, $b = 0$, $c = -k$ and $b^2 - 4ac = 4k$. $k > 0$: positive discriminant; $k < 0$: negative discriminant; $k = 0$: 0 discriminant.

55. The discriminant tells how many real solutions there are. If an equation looks difficult to solve, first use the discriminant to find out if it has any real solutions. If it doesn't, stop right there.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
30–32	1
33–35	2
36–38	3
39	4
40–42	5

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

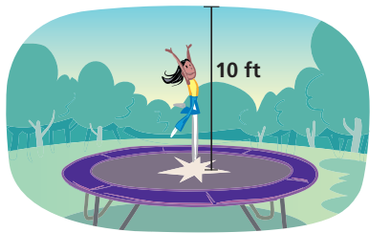
Solve using the Quadratic Formula.

30. $3x^2 = 13x - 4$ $\frac{1}{3}, 4$
31. $x^2 - 10x + 9 = 0$ 1, 9
32. $1 = 3x^2 + 2x$ $-1, \frac{1}{3}$
33. $x^2 - 4x + 1 = 0$
34. $3x^2 - 5 = 0$
35. $2x^2 + 7x = -4$

Find the number of real solutions of each equation using the discriminant.

36. $3x^2 - 6x + 3 = 0$ 1
37. $x^2 - 3x - 8 = 0$ 2
38. $7x^2 + 6x + 2 = 0$ 0

39. **Multi-Step** A gymnast who can stretch her arms up to reach 6 feet jumps straight up on a trampoline. The height of her feet above the trampoline can be modeled by the equation $h = -16x^2 + 12x$, where x is the time in seconds after her jump. Do the gymnast's hands reach a height of 10 feet above the trampoline? Use the discriminant to explain. (*Hint:* Let $h = 10 - 6$, or 4.)



No; the discriminant of $4 = -16x^2 + 12x$ is negative.

Solve.

40. $x^2 + 4x + 3 = 0$ $-3, -1$
41. $x^2 + 2x = 15$ $-5, 3$
42. $x^2 - 12 = -x$ $-4, 3$

Write each equation in standard form. Use the discriminant to determine the number of solutions. Then find any real solutions.

43. $2x = 3 + 2x^2$ no real solutions
44. $x^2 = 2x + 9$ 2 solutions; $\frac{2 - \sqrt{40}}{2} \approx -2.16$, $\frac{2 + \sqrt{40}}{2} \approx 4.16$
45. $2 = 7x + 4x^2$ 2 solutions; $-2, \frac{1}{4}$
46. $-7 = x^2$ no real solutions
47. $-12x = -9x^2 - 4$ 1 solution; $\frac{2}{3}$
48. $x^2 - 14 = 0$ 2 solutions; $\pm\sqrt{14} \approx \pm 3.74$

Multi-Step **Use the discriminant to determine the number of x-intercepts. Then find them.**

49. $y = 2x^2 - x - 21$ 2 x-intercepts; $\frac{7}{2}, -3$
50. $y = 5x^2 + 12x + 8$ no x-intercepts
51. $y = x^2 - 10x + 25$ 1 x-intercept; 5

52. Copy and complete the table.

Quadratic Equation	Discriminant	Number of Real Solutions
$x^2 + 12x - 20 = 0$	224	2
$8x + x^2 = -16$	0	1
$0.5x^2 + x - 3 = 0$	7	2
$-3x^2 - 2x = 1$	−8	0

53. **Sports** A diver begins on a platform 10 meters above the surface of the water. The diver's height is given by the equation $h(t) = -4.9t^2 + 3.5t + 10$, where t is the time in seconds after the diver jumps.
   a. How long does it take the diver to reach a point 1 meter above the water? about 1.76 s
   b. How many solutions does the equation you used in part **a** have? 2 solutions
   c. Do all of the solutions to the equation make sense in the situation? Explain. No; negative solutions do not make sense because the variable represents time.

54. **Critical Thinking** How many real solutions does the equation $x^2 = k$ have when $k > 0$, when $k < 0$, and when $k = 0$? Use the discriminant to explain.

55. **Write About It** How can you use the discriminant to save time?

PRACTICE A
PRACTICE C
PRACTICE B

Name _____ Date _____ Class _____

LESSON 8-9 **Practice B**
The Quadratic Formula and the Discriminant

Solve using the quadratic formula.

1. $x^2 + x = 12$ — 3, −4
2. $4x^2 - 17x - 15 = 0$ — 5, $-\frac{3}{4}$
3. $2x^2 - 5x = 3$ — 3, $-\frac{1}{2}$
4. $3x^2 + 14x - 5 = 0$ — $\frac{1}{3}$, −5

Find the number of real solutions of each equation using the discriminant.

5. $x^2 + 25 = 0$ — no real solutions
6. $x^2 - 11x + 28 = 0$ — 2
7. $x^2 + 8x + 16 = 0$ — 1

Solve using any method.

8. $x^2 + 8x + 15 = 0$ — −3, −5
9. $x^2 - 49 = 0$ — 7, −7
10. $6x^2 + x - 1 = 0$ — $\frac{1}{3}, -\frac{1}{2}$
11. $x^2 + 8x - 20 = 0$ — −10, 2

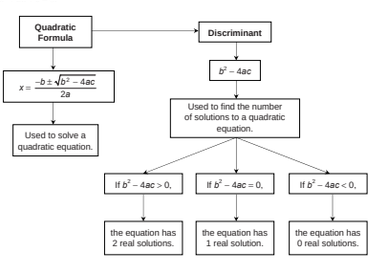
12. In the past, professional baseball was played at the Astrodome in Houston, Texas. The Astrodome has a maximum height of 63.4 m. The height of a baseball t seconds after it is hit straight up in the air with a velocity of 45 ft/s is given by $h = -9.8t^2 + 45t + 1$. Will a

READING STRATEGIES

Name _____ Date _____ Class _____

LESSON 8-9 **Reading Strategies**
Use a Graphic Organizer

This graphic organizer will help you keep track of the quadratic formula and its discriminant, which can be applied to quadratic equations in the form $ax^2 + bx + c = 0$.



Use the graphic organizer help you answer each question.

1. If the discriminant of a quadratic equation is −6, how many real solutions does it have? 0
2. Identify a, b, and c in the equation $2x^2 - 5x + 3 = 0$. $a = 2$ $b = -5$ $c = 3$
3. How many real solutions does the equation in problem 2 have? 2

Solve each quadratic equation using the quadratic formula.

4. $x^2 - 7x + 10 = 0$ — $x = 2$ or $x = 5$
5. $3x^2 + 4 = 0$ — no real solution
6. $3x^2 + x = 10$
7. $x^2 = 16x - 64$

RETEACH

Name _____ Date _____ Class _____

LESSON 8-9 **Review for Mastery**
The Quadratic Formula and the Discriminant

The Quadratic Formula can be used to solve any quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve $2x^2 - 5x - 12 = 0$ using the quadratic formula.

$2x^2 - 5x - 12 = 0$

Step 1: Identify a, b, and c.
$a = 2$
$b = -5$
$c = -12$

Step 2: Substitute into the quadratic formula.
$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-12)}}{2(2)}$

Step 3: Simplify.
$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-12)}}{2(2)}$
$x = \frac{5 \pm \sqrt{25 - (-96)}}{4}$
$x = \frac{5 \pm \sqrt{121}}{4}$
$x = \frac{5 \pm 11}{4}$

Step 4: Write two equations and solve.
$x = \frac{5 + 11}{4}$ or $x = \frac{5 - 11}{4}$
$x = 4$ or $x = -\frac{3}{2}$

Solve using the quadratic equation by filling in the blanks below.

1. $x^2 + 2x - 35 = 0$
   $a = 1$; $b = 2$; $c = -35$
   $x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-35)}}{2(1)}$
   Simplify: −7, 5
2. $3x^2 + 7x + 2 = 0$
   $a = 3$; $b = 7$; $c = 2$
   $x = \frac{-(7) \pm \sqrt{(7)^2 - 4(3)(2)}}{2(3)}$
   Simplify: $-\frac{1}{3}, -2$
3. $x^2 + x - 20 = 0$
   $a = 1$; $b = 1$; $c = -20$
   $x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-20)}}{2(1)}$
4. $2x^2 - 9x - 5 = 0$
   $a = 2$; $b = -9$; $c = -5$
   $x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(-5)}}{2(2)}$

MULTI-STEP TEST PREP

56. The equation $0 = -16t^2 + 80t + 20$ gives the time t in seconds when a golf ball is at height 0 feet.

a. Will the height of the ball reach 130 feet? Explain.

b. Will the golf ball reach a height of 116 feet? If so, when? **yes; 2 s and 3 s**

c. Solve the given quadratic equation using the Quadratic Formula. **≈ 5.24 s**

TEST PREP

57. How many real solutions does $4x^2 - 3x + 1 = 0$ have?

(A) 0 (B) 1 (C) 2 (D) 4

58. For which of the following conditions does $ax^2 + bx + c = 0$ have two real solutions?

I. $b^2 = 4ac$

II. $b^2 > 4ac$

III. $a = b, c = b$

(F) I only (G) II only (H) III only (J) II and III

59. Extended Response Use the equation $0 = x^2 + 2x + 1$ to answer the following.

a. How many solutions does the equation have? **1**

b. Solve the equation by graphing. **−1**

c. Solve the equation by factoring. **−1**

d. Solve the equation by using the Quadratic Formula. **−1**

e. Explain which method was easiest for you. Why?

59e. Possible answer: The easiest was solving by factoring: $0 = (x + 1)(x + 1)$ because the equation includes a perfect-square trinomial.

CHALLENGE AND EXTEND

60. **Agriculture** A rancher has 80 yards of fencing to build a rectangular pen. Let w be the width of the pen. Write an equation giving the area of the pen. Find the dimensions of the pen when the area is 400 square yards.

60. Area of pen $= 40w - w^2$; the width and length are each 20 yd.

61. **Agriculture** A farmer wants to fence a four-sided field using an existing fence along the south side of the field. He has 1000 feet of fencing. He makes the northern boundary perpendicular to and twice as long as the western boundary. The eastern and western boundaries have to be parallel, but the northern and southern ones do not.

a. Can the farmer enclose an area of 125,000 square feet? Explain why or why not. (*Hint:* Use the formula for the area of a trapezoid, $A = \frac{1}{2}h(b_1 + b_2)$.)

b. What geometric shape will the field be? **rectangle**

61a. Yes; the three boundaries can be represented by x, $2x$, and $1000 - 3x$. The area is represented by $\frac{1}{2}(2x)(x + 1000 - 3x)$, or $-2x^2 + 1000x$. So $-2x^2 + 1000x = 125{,}000$. Rewrite this in standard form: $2x^2 - 1000x + 125{,}000 = 0$. The discriminant is 0, so there is exactly one solution.

(tl),PhotoDisc/Getty Images; (tc),©COMSTOCK, Inc.

TEST PREP DOCTOR In **Exercise 58,** students can write $b^2 - 4ac > 0$ and rearrange it, yielding $b^2 > 4ac$. Thus, **II** is true and choices **F** and **H** can be eliminated. To find out whether **III** is true, substitute b for a and c in $b^2 > 4ac$. This yields $b^2 > 4b^2$, which simplifies to $1 > 4$, so **III** is false, and **J** can be eliminated.

Answers

56a. No; the discriminant of $130 = -16t^2 + 80t + 20$ is negative, so the equation has no real solutions.

Journal

Have students write about their preferred method of solving quadratic equations. Have them describe its advantages and disadvantages.

ALTERNATIVE ASSESSMENT

Have students write a quadratic equation and use the discriminant to find the number of real solutions. Have them use the Quadratic Formula to find the real solution(s), if any exist. If there are no real solutions, have them explain what this means.

Power Presentations with PowerPoint®

8-9 Lesson Quiz

1. Solve $-3x^2 + 5x = 1$ by using the Quadratic Formula.
$\frac{5 - \sqrt{13}}{6} \approx 0.23$,
$\frac{5 + \sqrt{13}}{6} \approx 1.43$

2. Find the number of real solutions of $5x^2 - 10x - 8 = 0$ by using the discriminant. 2

3. The height h in feet of an object shot straight up is modeled by $h = -16t^2 + vt + c$, where c is the beginning height of the object above the ground. An object is shot up from 4 feet off the ground with an initial velocity v of 48 feet per second. Will it reach a height of 40 feet? Use the discriminant to explain your answer. The discriminant is zero. The object will reach a height of 40 feet once.

4. Solve $8x^2 - 13x - 6 = 0$.
$2, -\frac{3}{8}$

Also available online

PROBLEM SOLVING

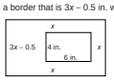
Name _____ Date _____ Class _____

LESSON 8-9 **Problem Solving**
The Quadratic Formula and the Discriminant

Write the correct answer.

1. Theo's flying disc got stuck in a tree 14 feet from the ground. Theo threw his shoe up at the disc to dislodge it. The height in feet h of the shoe is given by the equation $h = -16t^2 + 25t + 6$, where t is the time in seconds. Determine whether the shoe hit the disc. Use the discriminant to explain your answer.
Yes; the discriminant (113) is positive.

2. A picture frame holds a 4-in. by 6-in. photograph. The frame adds a border x inches wide around three sides of the photo. On the fourth side the frame forms a border that is $3x - 0.5$ in. wide.



The combined area of the photograph and the frame is 80.5 in^2. Write a quadratic equation for the combined area. Then use the quadratic formula to find x.
$8x^2 + 27x + 22 = 80.5$
$x = 1.5$

3. The manager of a park enclosed an area for small dogs to play. He made the length 15 feet longer than the width and enclosed an area covering 1350 square feet. What are the dimensions of the dogs' play area?
30 ft by 45 ft

The equation $-5x^2 + 72x + 378$ models the number of students enrolled in a school where x is the number of years since the school first opened in 1990. Select the best answer.

4. How many students did the school have when it opened?
A 68
B 72
C 378
D 445

5. Which equation can be used to find the year in which 502 students were enrolled?
F $-5x^2 + 72x + 502 = 0$
G $-5x^2 + 72x - 124 = 0$
H $-5x^2 + 72x - 502 = 0$
J $-5x^2 + 72x + 124 = 0$

6. In which year were 502 students enrolled?
A 1992 C 1998
B 1996 D 2002

7. In which year were 598 students enrolled?
F 1995 H 2000
G 1998 J 2010

8. Which statement is true?
A Enrollment exceeded 650 students at one point.

CHALLENGE

Name _____ Date _____ Class _____

LESSON 8-9 **Challenge**
Solving Quadratic Equations

The general form of a quadratic equation is $ax^2 + bx + 5 = 0$, where $a \neq 0$.

Identify a, b, and c for each of the following quadratic equations.

1. $2x^2 - 5x - 8 = 0$ — $a = 2; b = -5; c = -8$
2. $7x^2 - 56 = 0$ — $a = 7; b = 0; c = -56$
3. $12x^2 - 16x = 0$ — $a = 12; b = -16; c = 0$
4. Why is $5x - 15 = 0$ not a quadratic equation? $a = 0$; there is no x^2-term

Three methods for solving quadratic equations are taking square roots, factoring, and using the quadratic formula. Although the quadratic formula can be used to solve any quadratic equation, often one of the other methods is more efficient.

Use Exercises 1–3 to answer the following questions:

5. For which exercise would *taking square roots* be the quickest way to find a solution? 2
6. For which exercise would *factoring* be the quickest way to find a solution? 3
7. For which exercise would *using the quadratic formula* be the quickest way to find a solution? 1

Complete the following table to help you organize the results from the above exercises:

	Example	Method
if $b = 0$ and $c = 0$	$5x^2 = 0$	none; $x = 0$
8. if $b = 0$ and $c \neq 0$	answers may vary	answers may vary
9. if $b \neq 0$ and $c = 0$	answers may vary	answers may vary
10. if $b \neq 0$ and $c \neq 0$	answers may vary	answers may vary

11. Solve $ax^2 = 0$ for x. $x = 0$
12. Solve $ax^2 - c = 0$ for x in terms of a and c. $x = \pm\sqrt{\frac{c}{a}}$
13. Solve $ax^2 + bx = 0$ for x in terms of a and b. $x = \frac{-b}{a}$ or $x = 0$
14. Solve $ax^2 + bx + c = 0$ for x in terms of a, b and c. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

8-10 Organizer

Pacing: Traditional 2 days
Block 1 day

Objectives: Solve systems of equations in two variables in which one equation is linear and the other is quadratic.

Online Edition
Tutorial Videos

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Warm Up

Solve each quadratic equation by factoring. Check your answer.

1. $x^2 - 3x - 10 = 0$ 5, −2

2. $-3x^2 - 12x = 12$ −2

Find the number of real solutions of each equation using the discriminant.

3. $25x^2 - 10x + 1 = 0$ one

4. $2x^2 + 7x + 2 = 0$ two

5. $3x^2 + x + 2 = 0$ none

Also available online

Math Humor

Q: A parabola and a line got married and had two daughters. What were they?

A: Nonlinear sisters.

COMMON CORE

CC.9-12.A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

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COMMON CORE

8-10 Nonlinear Systems

CC.9-12.A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Objective
Solve systems of equations in two variables in which one equation is linear and the other is quadratic.

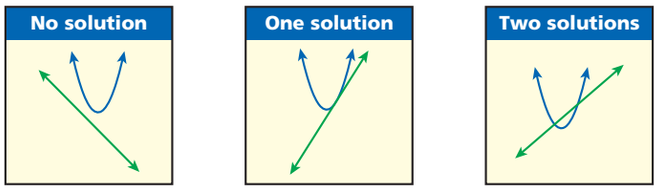
Vocabulary
nonlinear system of equations

Why learn this?
You can solve a nonlinear system to find how long it takes for two objects to reach the same height. (See Example 4.)

Recall that a system of linear equations is a set of two or more linear equations. A solution of a system is an ordered pair that satisfies each equation in the system. Points where the graphs of the equations intersect represent solutions of the system.

A **nonlinear system of equations** is a system in which at least one of the equations is nonlinear. For example, a system that contains one quadratic equation and one linear equation is a nonlinear system.

A system made up of a linear equation and a quadratic equation can have no solution, one solution, or two solutions, as shown below.



EXAMPLE 1 **Solving a Nonlinear System by Graphing**

Solve the system by graphing. Check your answer.

$$\begin{cases} y = x^2 - 2x - 3 \\ y = -x - 1 \end{cases}$$

Step 1 Graph $y = x^2 - 2x - 3$.

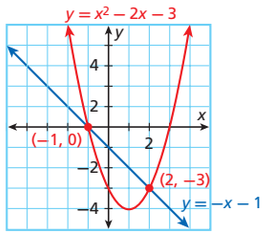
The axis of symmetry is $x = 1$.

The vertex is $(1, -4)$.

The y-intercept is -3.

Another point is $(-1, 0)$.

Graph the points and reflect them across the axis of symmetry.



Step 2 Graph $y = -x - 1$.

The slope is -1.

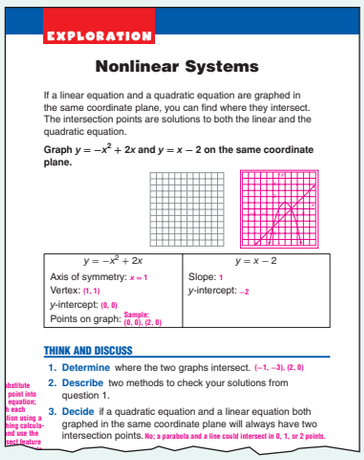
The y-intercept is -1.

Step 3 Find the points where the two graphs intersect.

The solutions appear to be $(-1, 0)$ and $(2, -3)$.

Remember!
A quadratic function has the form $y = ax^2 + bx + c$. To graph a quadratic function, start by using $x = -\frac{b}{2a}$ to find the axis of symmetry and the vertex.

1 Introduce


EXPLORATION

Nonlinear Systems

If a linear equation and a quadratic equation are graphed in the same coordinate plane, you can find where they intersect. The intersection points are solutions to both the linear and the quadratic equation.

Graph $y = -x^2 + 2x$ and $y = x - 2$ on the same coordinate plane.

$y = -x^2 + 2x$	$y = x - 2$
Axis of symmetry: $x = 1$	Slope: 1
Vertex: (1, 1)	y-intercept: −2
y-intercept: (0, 0)	
Points on graph: Sample: (0, 0), (2, 0)	

THINK AND DISCUSS

1. **Determine** where the two graphs intersect. (−1, −3), (2, 0)
2. **Describe** two methods to check your solutions from question 1. Substitute point into equation; graph each equation using a graphing calculator and use the intersect feature
3. **Decide** if a quadratic equation and a linear equation both graphed in the same coordinate plane will always have two intersection points. No; a parabola and a line could intersect in 0, 1, or 2 points.

Motivate

Remind students that they have learned to solve linear systems by graphing, by substitution, and by elimination. Remind them that they have also learned to solve quadratic equations by graphing, by factoring, and by the Quadratic Formula. Tell them that in this lesson they will use all of those skills to solve systems made up of a linear equation and a quadratic equation.

Explorations and answers are provided in your online resources.

Check

Substitute $(-1, 0)$ into the system.

$y = x^2 - 2x - 3$	
0	$(-1)^2 - 2(-1) - 3$
0	$1 + 2 - 3$
0	0 ✓

$y = -x - 1$	
0	$-(-1) - 1$
0	$1 - 1$
0	0 ✓

Substitute $(2, -3)$ into the system.

$y = x^2 - 2x - 3$	
-3	$2^2 - 2(2) - 3$
-3	$4 - 4 - 3$
-3	-3 ✓

$y = -x - 1$	
-3	$-2 - 1$
-3	-3 ✓

The solutions are $(-1, 0)$ and $(2, -3)$.

CHECK IT OUT! **1.** Solve the system by graphing. Check your answer.

$$\begin{cases} y = x^2 - 4x + 5 \\ y = x + 1 \end{cases}$$ **(1, 2), (4, 5)**

EXAMPLE 2 Solving a Nonlinear System by Substitution

Solve the system by substitution.

$$\begin{cases} y = 2x^2 - 3x + 4 \\ y = x + 2 \end{cases}$$

Remember!

The substitution method is a good choice when either equation is solved for a variable, both equations are solved for the same variable, or a variable in either equation has a coefficient of 1 or −1.

$y = 2x^2 - 3x + 4$ — *Both equations are solved for y.*

$y = x + 2$

$y = 2x^2 - 3x + 4$

$x + 2 = 2x^2 - 3x + 4$ — *Substitute $x + 2$ for y in the first equation.*

$-(x + 2) \quad -(x + 2)$ — *Subtract $x + 2$ from both sides.*

$0 = 2x^2 - 4x + 2$

$0 = 2(x^2 - 2x + 1)$ — *Factor out the GCF, 2.*

$0 = 2(x - 1)(x - 1)$ — *Factor the trinomial.*

$2 \neq 0; \quad x - 1 = 0$ — *Use the Zero Product Property; 2 cannot equal 0.*

$x = 1$ — *Solve the remaining equation.*

$y = x + 2$ — *Write one of the original equations.*

$y = 1 + 2$ — *Substitute 1 for x.*

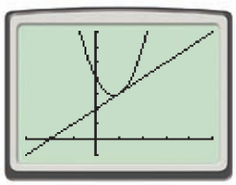
$y = 3$

The solution is $(1, 3)$.

Check

Use a graphing calculator.

The graph supports the result found above. This system has exactly one real solution. The graph of the system consists of a line and a parabola that meet in exactly one point.



CHECK IT OUT! **2.** Solve the system by substitution. Check your answer.

$$\begin{cases} y = 3x^2 - 3x + 1 \\ y = -3x + 4 \end{cases}$$ **(−1, 7), (1, 1)**

COMMON ERROR ALERT

When solving a system, students sometimes stop after finding the value(s) of one of the variables. Remind them that solutions of a system are ordered pairs; they need to substitute the value(s) they found into one of the original equations and solve for the other variable.

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Additional Examples

Example 1

Solve the system by graphing. Check your answer.

$$\begin{cases} y = x^2 + 4x + 3 \\ y = x + 3 \end{cases}$$ **(−3, 0), (0, 3)**

Example 2

Solve the system by substitution.

$$\begin{cases} y = x^2 - x - 5 \\ y = -3x + 3 \end{cases}$$ **(2, −3), (−4, 15)**

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- To graph the quadratic equation, how do you find the axis of symmetry and the vertex?
- Why should you always check your answers when solving by graphing?

EXAMPLE 2

- When solving a linear/quadratic system by substitution, why is it easier to solve for x first?

Teaching Tip **Technology** Encourage students to use a graphing calculator to check that their answers are reasonable.

2 Teach

Guided Instruction

Review the three possible cases when solving a linear system: one solution (intersecting lines), infinitely many solutions (coincident lines), and no solution (parallel lines). Then go over the three possible cases for linear/quadratic systems. Remind students that they have already learned the individual skills and concepts needed in this lesson, and they need only combine them.

Reaching All Learners

Through Visual Cues

When multiplying is necessary in the elimination method, have students use color and labels to identify the equations and their processes, as shown below for **Example 3B**.

Eq. 1 $y = x^2 + x - 1$

Eq. 2 $2x - 3y = 6$

Eq. 1 $3y = 3x^2 + 3x - 3$ ← Multiply each term by 3.

Eq. 2 $2x - 3y = 6$

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Additional Examples

Example 3

Solve each system by elimination.

A. $\begin{cases} 3x - y = 1 \\ y = x^2 + 4x - 7 \end{cases}$

(2, 5); (−3, −10)

B. $\begin{cases} y = 2x^2 + x - 1 \\ x - 2y = 6 \end{cases}$ no sol.

Also available online

Questioning Strategies

EXAMPLE 3

- How can you tell that elimination is a good method for solving a given system?
- When multiplication is needed, how do you know what to multiply by to eliminate *y*?

Math Background

Remind students why a negative discriminant indicates that there are no real solutions: A negative number has no real square root. For example, $\sqrt{25} = 5$ because $5^2 = 25$. However, $\sqrt{-25}$ is not a real number because there is no real number that you can square to obtain −25.

EXAMPLE 3 Solving a Nonlinear System by Elimination

Solve each system by elimination.

A $\begin{cases} 4x - y = 2 \\ y = x^2 + 1 \end{cases}$

$4x - y = 2$ — *Write the system to align the y-terms.*

$+ \quad y = x^2 + 1$ — *Add the equations to eliminate y.*

$4x = x^2 + 3$

$-4x \quad -4x$ — *Subtract 4x from both sides.*

$0 = x^2 - 4x + 3$

$0 = (x - 1)(x - 3)$ — *Factor the trinomial.*

$x - 1 = 0$ or $x - 3 = 0$ — *Use the Zero Product Property.*

$x = 1$ or $x = 3$ — *Solve the equations.*

$y = x^2 + 1 \qquad y = x^2 + 1$ — *Write one of the original equations.*

$y = 1^2 + 1 \qquad y = 3^2 + 1$ — *Substitute each x-value and solve for y.*

$y = 2 \qquad y = 10$

The solutions are (1, 2) and (3, 10).

B $\begin{cases} y = x^2 + x - 1 \\ 2x - 3y = 6 \end{cases}$

$y = x^2 + x - 1$ — *Write the system to align the y-terms.*

$2x - 3y = 6$

$3(y) = 3(x^2 + x - 1)$ — *Multiply each term in the first equation by 3.*

$2x - 3y = 6$

$3y = 3x^2 + 3x - 3$

$+ \; 2x - 3y = 6$ — *Add the second equation to the new first equation to eliminate y.*

$2x = 3x^2 + 3x + 3$

$0 = 3x^2 + x + 3$ — *Subtract 2x from both sides.*

$x = \dfrac{-1 \pm \sqrt{(1)^2 - 4(3)(3)}}{2(3)}$ — *Use the Quadratic Formula,* $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$x = \dfrac{-1 \pm \sqrt{1 - 36}}{6}$

$x = \dfrac{-1 \pm \sqrt{-35}}{6}$ — *Note the discriminant: $b^2 - 4ac = -35$. Its value is negative, so there are no real solutions.*

There are no real solutions.

Remember!

The elimination method is a good choice when both equations have the same variable term with the same or opposite coefficients or when a variable term in one equation is a multiple of the corresponding variable term in the other equation.

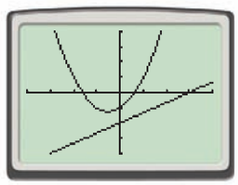
Check Use a graphing calculator.

To graph $2x - 3y = 6$, first solve for *y*.

$2x - 3y = 6$

$-3y = -2x + 6$

$y = \frac{2}{3}x - 2$



The graph supports that there are no real solutions.

CHECK IT OUT! Solve each system by elimination. Check your answers.

3a. $\begin{cases} 2x - y = 2 \\ y = x^2 - 5 \end{cases}$ (3, 4), (−1, −4)

3b. $\begin{cases} y = x^2 - 2x - 5 \\ 5x - 2y = 5 \end{cases}$ (5, 10), (−0.5, −3.75)

EXAMPLE 4 *Physics Application*

An elevator is rising at a constant rate of 20 feet per second. Its height in feet after t seconds is given by $h = 20t$. At the instant the elevator is at ground level, a ball is thrown upward with an initial velocity of 80 feet per second from ground level. The height in feet of the ball after t seconds is given by $h = -16t^2 + 80t$. Find the time it takes for the ball and the elevator to reach the same height.

Helpful Hint

When $t = 0$, the ball and elevator are at the same height because they are both at ground level.

Solve the system $\begin{cases} h = -16t^2 + 80t \\ h = 20t \end{cases}$ by substitution.

$-16t^2 + 80t = 20t$ *Substitute $-16t^2 + 80t$ for h in the second equation.*

$-20t \quad -20t$ *Subtract $20t$ from both sides.*

$-16t^2 + 60t = 0$

$-4t(4t - 15) = 0$ *Factor out the GCF, $-4t$.*

$-4t = 0$ or $4t - 15 = 0$ *Use the Zero Product Property.*

$t = 0 \qquad 4t = 15$ *Solve the remaining equations.*

$t = 3.75$

It takes 3.75 seconds for the ball and the elevator to reach the same height.

CHECK IT OUT! **4.** An elevator is rising at a constant rate of 8 feet per second. Its height in feet after t seconds is given by $h = 8t$. At the instant the elevator is at ground level, a ball is dropped from a height of 120 feet. The height in feet of the ball after t seconds is given by $h = -16t^2 + 120$. Find the time it takes for the ball and the elevator to reach the same height. 2.5 s

THINK AND DISCUSS

1. How is solving the systems in this lesson similar to solving systems of linear equations? How is it different?

2. When using elimination to solve a linear/quadratic system, which variable will be eliminated? Why?

3. A system of linear equations can have infinitely many solutions. Why can't a linear/quadratic system have infinitely many solutions?

4. GET ORGANIZED Copy and complete the graphic organizer by sketching diagrams to show examples. Write *not possible* for any cases that are not possible.

System of Equations	Number of Solutions			
	0	1	2	infinite
Linear				
Linear/Quadratic				

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Additional Examples

Example 4

The increasing enrollment at South Ridge High School can be modeled by the equation $E(t) = -t^2 + 25t + 600$, where t represents the number of years after 2010. The increasing enrollment at Alta Vista High School can be modeled by the equation $E(t) = 24t + 570$. In what year will the enrollments at the two schools be equal? 2016

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 4

- Are the values of t solutions of the system of equations?
- If you found the complete solutions to the system, what would they tell you?

3 Close

Summarize

Point out that substitution or elimination can be used to solve any linear/quadratic system, but graphing, especially when done by hand, has limited usefulness when the coordinates of the solutions are not integers.

Ask students what the graph looks like for a linear/quadratic system that has:

- no solution no intersection
- one solution one point of intersection
- two solutions two points of intersection

ONGOING ASSESSMENT and INTERVENTION

***Diagnose* *Before* the Lesson**
Warm Up, TE

***Monitor* *During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess* *After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

Possible answers:

1. Possible answer: Similar: You can use graphing, substitution, and elimination. Different: When using substitution and elimination, you need to solve quadratic equations.
2. y; you cannot eliminate x because one equation has an x^2-term and the other equation does not.
3. A line and a parabola cannot coincide.
4. See Additional Answers.

8-10 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

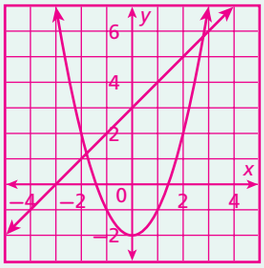
If you finished Examples **1–2**
- **Basic** 10–17, 26, 30
- **Average** 10–17, 26–33, 36
- **Advanced** 10–17, 26, 30, 36

If you finished Examples **1–4**
- **Basic** 10–26, 30, 33, 37–41
- **Average** 10–47
- **Advanced** 10–26, 30, 34–47

Homework Quick Check
Quickly check key concepts.
Exercises: 10, 16, 18, 22, 30

Answers

2. Possible answer:



MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 9, 22–25, 47

Construct viable arguments and critique the reasoning of others. Exercises 34–35, 37

Attend to precision. Exercise 36

Look for and make use of structure. Exercise 46

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GUIDED PRACTICE

Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. A system of equations that includes a linear equation and a quadratic equation is _____?_____. (*linear*, *nonlinear*, or *quadratic*) **nonlinear**
2. Sketch a nonlinear system of equations that has two solutions. The system should include one quadratic equation and one linear equation.

SEE EXAMPLE 1

Solve each system by graphing. Check your answers.

3. $\begin{cases} y = 2x^2 - 7x + 6 \\ y = x \end{cases}$ **(1, 1), (3, 3)**

4. $\begin{cases} y = x^2 - 2x - 5 \\ y = 2x - 8 \end{cases}$ **(1, −6), (3, −2)**

SEE EXAMPLE 2

Solve each system by substitution. Check your answers.

5. $\begin{cases} y = x^2 - 4x + 3 \\ y = x - 3 \end{cases}$ **(3, 0), (2, −1)**

6. $\begin{cases} y = 2x^2 - 5x + 3 \\ y = -3x + 15 \end{cases}$ **(−2, 21), (3, 6)**

SEE EXAMPLE 3

Solve each system by elimination. Check your answers.

7. $\begin{cases} y = x^2 - 3 \\ 4x - y = 6 \end{cases}$ **(1, −2), (3, 6)**

8. $\begin{cases} y = x^2 + 7x + 12 \\ 3x - y = 5 \end{cases}$ **no sol.**

SEE EXAMPLE 4

9. **Physics** A bird is flying upwards such that its height in feet after t seconds is given by $h = 4t$. At the instant the bird passes the height of a ball being held out of a window, the ball is thrown upward with an initial velocity of 80 feet per second. The height in feet of the ball after t seconds is given by $h = -16t^2 + 80t$. Find the time it takes for the ball and the bird to reach the same height. **4.75 s**

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
10–13	1
14–17	2
18–21	3
22–25	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Solve each system by graphing. Check your answers.

10. $\begin{cases} y = x^2 - 4 \\ y = 5x - 10 \end{cases}$ **(2, 0), (3, 5)**

11. $\begin{cases} y = x^2 - 3 \\ x - 6y = 18 \end{cases}$ **(0, −3)**

12. $\begin{cases} y = x^2 + 4x + 7 \\ y = x + 5 \end{cases}$ **(−1, 4), (−2, 3)**

13. $\begin{cases} y = 2x^2 - 8x + 3 \\ y = 6x - 21 \end{cases}$ **(3, −3), (4, 3)**

Solve each system by substitution. Check your answers.

14. $\begin{cases} y = x^2 + 7x + 2 \\ y = 5x + 5 \end{cases}$ **(1, 10), (−3, −10)**

15. $\begin{cases} y = 2x^2 - 3 \\ y = 2x + 9 \end{cases}$ **(3, 15), (−2, 5)**

16. $\begin{cases} y = x^2 - 5 \\ y = -2x + 3 \end{cases}$ **(2, −1), (−4, 11)**

17. $\begin{cases} y = 5x^2 - 2x \\ y = 10x + 9 \end{cases}$ **(3, 39), $\left(-\frac{3}{5}, 3\right)$**

Solve each system by elimination. Check your answers.

18. $\begin{cases} y = 2x^2 - 3x + 1 \\ 5x - y = -1 \end{cases}$ **(4, 21), (0, 1)**

19. $\begin{cases} y = x^2 - 5 \\ x - 3y = 15 \end{cases}$ **(0, −5), $\left(\frac{1}{3}, -\frac{44}{9}\right)$**

20. $\begin{cases} y = 2x^2 - x + 7 \\ 2x + 3y = 6 \end{cases}$ **no sol.**

21. $\begin{cases} y = x^2 + 5x \\ 9x - y = 3 \end{cases}$ **(1, 6), (3, 24)**

22. **Demographics** The growing population of town A can be modeled by the equation $P(t) = 8t^2 + 2000$, where t represents number of years after 2010. The growing population of town B can be modeled by the equation $P(t) = 100t + 3000$. In which year will the populations of the towns be approximately equal? **2029**

23. **Finance** The value of Danielle's investments is modeled by the equation $V(t) = 3t^2 + 70t + 100$, where t represents the number of months after she made her initial investment. Jeffrey has no money invested in stocks, but he deposits the same amount every month into a savings account that he opened at the same time as Danielle began investing. His savings account balance can be modeled by the equation $V(t) = 50t + 275$. After how many months will the value of Danielle's investments be equal to the balance of Jeffrey's savings account? **5 mo**

24. **Amusement Parks** A ride at an amusement park consists of an observation deck that travels directly up into the air at a constant rate of 40 feet per second. Its height in feet after t seconds is given by $h = 40t$. At the instant the deck is at ground level, a ball is thrown up with initial velocity 60 feet per second from ground level. The height in feet of the ball after t seconds is given by $h = -16t^2 + 60t$. Find the time it takes for the ball and the deck to reach the same height. Round your answer to the nearest hundredth. **1.25 s**

25. **Business** A company's weekly revenue can be modeled by the equation $C(p) = 0.75p^2 + 10p + 200$, where p represents the number of products sold. The weekly cost of running the business is modeled by the equation $C(p) = 80p + 700$. How many products must the company sell in a week to break even (when revenue equal the costs of running the business)? **100**

Determine whether the point is a solution of the system of equations.

26. $\begin{cases} y = x^2 - 9x + 2 \\ 2x + y = -16 \end{cases}$; $(-2, -12)$ **no**

27. $\begin{cases} y = x^2 + 2x - 9 \\ y = 8x \end{cases}$; $(3, 24)$ **no**

28. $\begin{cases} y = x^2 - 6x - 1 \\ 3x - 4y = -3 \end{cases}$; $(7, 6)$ **yes**

29. $\begin{cases} y = 3x^2 - 7x + 6 \\ y = x + 1 \end{cases}$; $(1, 2)$ **yes**

30. $\begin{cases} y = 2x^2 - 5x + 5 \\ y = x + 5 \end{cases}$; $(3, 8)$ **yes**

31. $\begin{cases} y = 3x^2 + 4x - 1 \\ 9x - 2y = -5 \end{cases}$; $(-1, -2)$ **yes**

32. $\begin{cases} y = 2x^2 - 25 \\ 4x - y = 5 \end{cases}$; $(5, 25)$ **no**

33. $\begin{cases} y = x^2 + 3x + 8 \\ 5x - y = 28 \end{cases}$; $(2, -18)$ **no**

34. **Write About It** Explain in your own words when you should use the substitution method to solve a nonlinear system, and when you should use the elimination method.

35. **Critical Thinking** Describe a scenario in which you might use the graphing method to solve a system of nonlinear equations, even if you didn't expect the solution(s) to consist of integer coordinates.

36. **Estimation** Estimate the solution(s) to the system by graphing.

$\begin{cases} y = x^2 + 6x - 2 \\ y = 0.5x + 7 \end{cases}$ **Possible answer: (−7, 4); (1, 8)**

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Teaching Tip

Inclusion In **Exercise 25,** some students might be unfamiliar with the business terms *revenue, cost,* and *break even*. Explain that a company's *revenue* is the amount of money it makes, and the *cost* is the amount of money it spends. To *break even* means that the revenue and cost are equal.

Answers

34. Possible answer: Use substitution when one equation is already solved for a variable. Use elimination when the coefficients of the linear terms in both equations are opposites, or could be made opposites by integer multiplication.

35. When only an approximate answer is needed, or to check whether the answer found by another method is reasonable

TEST PREP DOCTOR

For **Exercise 38,** students who selected **B** most likely made a sign error when finding one of the *x*-values.

For **Exercise 39,** students who selected **J** verified that (2, 6) is a solution of the second equation, but did not verify that it is a solution of the first equation.

37. ///**ERROR ANALYSIS**/// Below are two solutions to the system of equations $\begin{cases} y = x^2 - 4x + 5 \\ 4x + 3y = 11 \end{cases}$. Which is incorrect? Explain the error.

B; the left side of the eq. was multiplied by −3, but the right side was not.

A

$y = x^2 - 4x + 5 \rightarrow \quad -3y = -3x^2 + 12x - 15$

$4x + 3y = 11 \quad \rightarrow 4x + 3y = 11$

$4x + 0 = -3x^2 + 12x - 4$

$0 = -3x^2 + 8x - 4$

$0 = (-3x + 2)(x - 2)$

$-3x + 2 = 0 \quad \text{or} \quad x - 2 = 0$

$-3x = -2 \qquad x = 2$

$x = \frac{2}{3}$

$4x + 3y = 11 \qquad 4x + 3y = 11$

$4\left(\frac{2}{3}\right) + 3y = 11 \qquad 4(2) + 3y = 11$

$\frac{8}{3} + 3y = 11 \qquad 8 + 3y = 11$

$3y = \frac{25}{3} \qquad 3y = 3$

$y = \frac{25}{9} \qquad y = 1$

The solutions are $\left(\frac{2}{3}, \frac{25}{9}\right)$ and (2, 1).

B

$y = x^2 - 4x + 5 \rightarrow \quad -3y = x^2 - 4x + 5$

$4x + 3y = 11 \quad \rightarrow 4x + 3y = 11$

$4x + 0 = x^2 - 4x + 5$

$0 = x^2 - 8x + 5$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(5)}}{2(1)}$

$x = \frac{8 \pm \sqrt{44}}{2}$

$x = \frac{8 \pm 2\sqrt{11}}{2}$

$x = 4 \pm \sqrt{11} \approx 7.32 \quad \text{or} \quad 0.68$

$4x + 3y \approx 11 \qquad 4x + 3y \approx 11$

$4(7.32) + 3y \approx 11 \qquad 4(0.68) + 3y \approx 11$

$29.28 + 3y \approx 11 \qquad 2.72 + 3y \approx 11$

$3y \approx -18.28 \qquad 3y \approx 8.28$

$y \approx -6.09 \qquad y \approx 2.76$

The solutions are approximately (7.32, −6.09) and (0.68, 2.76).

TEST PREP

38. Which are the solutions to the system of equations below?

$\begin{cases} y = x^2 - 5x - 2 \\ y = -7x + 1 \end{cases}$

Ⓐ (−3, 22) and (1, −6)
Ⓑ (−3, 22) and (−1, 8)
Ⓒ (−1, 8) and (2, −10)
Ⓓ (1, −6) and (2, −10)

39. For which system of equations is (2, 6) a solution?

Ⓕ $\begin{cases} y = x^2 + 4x - 2 \\ y = -3x - 2 \end{cases}$
Ⓗ $\begin{cases} y = 3x^2 - x - 4 \\ y = -5x + 16 \end{cases}$
Ⓖ $\begin{cases} y = x^2 - 4x + 14 \\ x - y = 4 \end{cases}$
Ⓙ $\begin{cases} y = 2x^2 + 9 \\ y = 5x - 4 \end{cases}$

40. Which system below has no real solutions?

Ⓐ $\begin{cases} 3y + x = 9 \\ y = (x - 3)^2 \end{cases}$
Ⓒ $\begin{cases} y = x - 4x^2 \\ x = -2 - y \end{cases}$
Ⓑ $\begin{cases} y = 2x^2 - 2 \\ x - y = 5 \end{cases}$
Ⓓ $\begin{cases} y = 2x - 7 \\ y = x^2 - 7 \end{cases}$

PRACTICE A

PRACTICE C

PRACTICE B

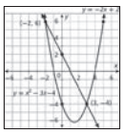
A *nonlinear system of equations* is a system in which at least one of the equations is nonlinear.

Possible Solutions for a Linear-Quadratic System

No Solutions	One Solution	Two Solutions

Solve the system by graphing. Check your answer.

$\begin{cases} y = x^2 - 3x - 4 \\ y = -2x + 2 \end{cases}$



Step 1: Graph $y = x^2 - 3x - 4$.
Axis of symmetry: $x = 1.5$; vertex: (1.5, −6.25)
y-intercept: (0, −4); another point (−2, 6)
Graph the points and reflect them across the axis of symmetry

Step 2: Graph $y = -2x + 2$.
Slope: −2; *y*-intercept: 2

Step 3: Find the points of intersection: (−2, 6) and (3, −4)

Check: Substitute the solutions into each system.

(−2, 6)	(3, −4)
$y = x^2 - 3x - 4$	$y = x^2 - 3x - 4$
$6 = (-2)^2 - 3(-2) - 4$	$-4 = 3^2 - 3(3) - 4$
$6 = 6$✓	$-4 = -4$✓
$y = -2x + 2$	$y = -2x + 2$
$6 = -2(-2) + 2$	$-4 = -2(3) + 2$
$6 = 6$✓	$-4 = -4$✓

Solve each system by graphing. Check your answers.

1. $\begin{cases} y = x^2 + 3x - 4 \\ y = 4x - 4 \end{cases}$ **(0, −4), (1, 0)**

2. $\begin{cases} y = 3x^2 + 2x - 1 \\ y = 2x + 2 \end{cases}$ **(−1, 0), (1, 4)**

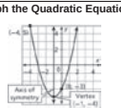
READING STRATEGIES

You can solve a nonlinear system of equations by graphing.

System of Equations

$\begin{cases} y = x^2 + 2x - 3 \\ y = 2x - 2 \end{cases}$

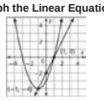
Graph the Quadratic Equation



Locate the axis of symmetry $x = -1$.
Locate key points to help you graph. Reflect points over the axis of symmetry to graph other points.

Nonlinear Systems

Graph the Linear Equation



Add the graph of $y = 2x - 2$.

Slope: 2 *y*-intercept: −2

Estimate the Solutions

Find the points where the two graphs intersect.
The solutions appear to be (−1, −4) and (1, 0).

Complete the following.

1. To graph a quadratic equation, first locate the **axis of symmetry** of ____. To graph a linear equation, find the **slope** and the ***y*-intercept** ____.

2. Solve the system by graphing.

$\begin{cases} y = x^2 - x - 6 \\ y = x - 3 \end{cases}$ **(−1, −4), (3, 0)**

RETEACH

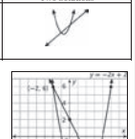
A *nonlinear system of equations* is a system in which at least one of the equations is nonlinear.

Possible Solutions for a Linear-Quadratic System

No Solutions	One Solution	Two Solutions

Solve the system by graphing. Check your answer.

$\begin{cases} y = x^2 - 3x - 4 \\ y = -2x + 2 \end{cases}$



Step 1: Graph $y = x^2 - 3x - 4$.
Axis of symmetry: $x = 1.5$; vertex: (1.5, −6.25)
y-intercept: (0, −4); another point (−2, 6)
Graph the points and reflect them across the axis of symmetry

Step 2: Graph $y = -2x + 2$.
Slope: −2; *y*-intercept: 2

Step 3: Find the points of intersection: (−2, 6) and (3, −4)

Check: Substitute the solutions into each system.

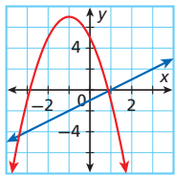
(−2, 6)	(3, −4)
$y = x^2 - 3x - 4$	$y = x^2 - 3x - 4$
$6 = (-2)^2 - 3(-2) - 4$	$-4 = 3^2 - 3(3) - 4$
$6 = 6$✓	$-4 = -4$✓
$y = -2x + 2$	$y = -2x + 2$
$6 = -2(-2) + 2$	$-4 = -2(3) + 2$
$6 = 6$✓	$-4 = -4$✓

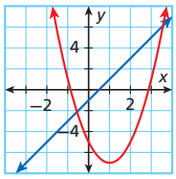
Solve each system by graphing. Check your answers.

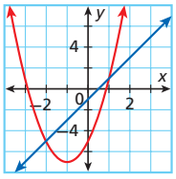
1. $\begin{cases} y = x^2 + 3x - 4 \\ y = 4x - 4 \end{cases}$ **(0, −4), (1, 0)**

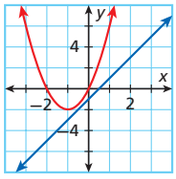
2. $\begin{cases} y = 3x^2 + 2x - 1 \\ y = 2x + 2 \end{cases}$ **(−1, 0), (1, 4)**

41. Which is the graph of $\begin{cases} y = 2x - 1 \\ y = 2x^2 + 4x - 5 \end{cases}$?

Ⓕ 

Ⓗ 

Ⓖ 

Ⓙ 

CHALLENGE AND EXTEND

Find the *x*-coordinate(s) of the solution(s) of each system.

42. $\begin{cases} y = 3x^2 + 4x - 7 \\ 3x + 5y = 8 \end{cases}$ **43.** $\begin{cases} y = x^2 - 3x - 5 \\ 3x + 2y = 8 \end{cases}$ **44.** $\begin{cases} y = x^2 + x - 2 \\ 3x + 2y = 3 \end{cases}$ **45.** $\begin{cases} y = 4x^2 - 9 \\ y = 7x - 2 \end{cases}$

46. A system of two equations contains one quadratic equation and one linear equation. The quadratic equation in the system is $y = x^2 + 5x - 9$. The solutions of the system are (3, 15) and (−1, −13). What is the linear equation in the system? **$y = 7x - 6$**

47. **Physics** The formula for the height of an object in free fall (neglecting air resistance) is $h(t) = -16t^2 + v_0t + h_0$, where v_0 is the object's initial velocity in feet per second and h_0 is the object's initial height above the ground in feet. One ball is thrown with an initial velocity of 90 ft/s from a height of 20 ft. A second ball is thrown at the exact same instant with an initial velocity of 80 ft/s and a height of 30 ft. After how many seconds will the balls reach the same height? **1 s**

42. $\frac{23 \pm \sqrt{3109}}{-30}$

43. $\frac{-3 \pm \sqrt{153}}{-4}$

44. $1, -\frac{7}{2}$

45. $\frac{7 \pm \sqrt{161}}{8}$

Journal

Have students explain how to check a solution of a linear/quadratic system algebraically. Have them also explain how to use a graphing calculator to check that they have the correct number of solutions.

ALTERNATIVE ASSESSMENT

Ask students to explain how to solve this system by graphing and how to solve it by substitution:

$\begin{cases} y = x^2 - 4 \\ y = x - 2 \end{cases}$

Ask students to explain how to solve this system by elimination:

$\begin{cases} y = 2x^2 - x + 1 \\ 3x - y = -1 \end{cases}$

Power Presentations with PowerPoint®

8-10 Lesson Quiz

Solve each system by the indicated method.

1. Graphing: $\begin{cases} y = x^2 - 4x + 3 \\ y = x - 1 \end{cases}$
(1, 0), (4, 3)

2. Substitution: $\begin{cases} y = 2x^2 - 9x - 5 \\ y = -3x + 3 \end{cases}$
(−1, 6), (4, −9)

3. Elimination: $\begin{cases} y = x^2 + 2x - 3 \\ x - y = 5 \end{cases}$
no solution

4. Elimination: $\begin{cases} y = x^2 - 7x + 10 \\ 2x - y = 8 \end{cases}$
(3, −2), (6, 4)

Also available online

PROBLEM SOLVING

Write the correct answer.

1. A ball is thrown upward with an initial velocity of 40 feet per second from ground level. The height *h* in feet of the ball after *t* seconds is given by $h = -16t^2 + 40t$. At the same time, a balloon is rising at a constant rate of 10 feet per second. Its height *h* in feet after *t* seconds is given by $h = 10t$. Find the time it takes for the ball and the balloon to reach the same height.
1.875 s

2. A bird starts flying up from the grass in a park and climbs at a steady rate of 0.5 meters per second. Its height *h* in meters after *t* seconds is given by $h = 0.5t$. The equation $h = -4.9t^2 + 40t + 3$ models the height *h*, in meters, of a baseball *t* seconds after it is hit. Find the approximate time it takes for the ball and the bird to reach the same height.
8.14 s

3. A skateboard company's monthly sales income can be modeled by the equation $C(s) = 0.5s^2 + 25s + 500$, where *s* represents the number of skateboards sold. The monthly cost of running the business is $C(s) = 25s + 812.5$. How many skateboards must the company sell in a month before the sales income equals or exceeds the cost of running the business?
25

4. The deer population in a park can be modeled by the equation $P(y) = 4y^2 - 10y + 60$, where *y* is the number of years after 2010. The deer population in another park can be modeled by $P(y) = 10y + 80$, where *y* is the number of years after 2010. In which year will the two parks have approximately the same number of deer?
2016

Select the best answer.

5. A seagull is flying upwards such that its height *h* in feet above sea level after *t* seconds is given by $h = 3t$. At the same time, the height *h* in feet of a rock falling off a cliff above the sea after *t* seconds is given by $h = -16t^2 + 50$. Find the approximate time it takes for the rock and the bird to be at the same height.

A 1.68 s C 3.36 s
B 3.13 s D 16.67 s

A

6. A juggler at a circus throws a ball upwards such that the ball's height *h* in feet above the ground after *t* seconds is given by $h = -16t^2 + 20t + 5$. At the same time, a scenic elevator begins climbing a tower at a constant rate of 20 feet per second. Its height *h* in feet after *t* seconds is given by $h = 20t$. Find the approximate time it takes for the ball and the elevator to reach the same height.

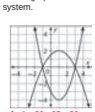
F 0.56 s H 4 s
G 1.12 s J 11.18 s

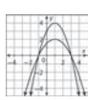
F

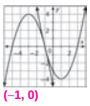
CHALLENGE

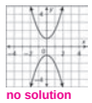
Another nonlinear system of equations in which both equations are quadratic is called a *quadratic-quadratic system*. The graph of a quadratic-quadratic system consists of two parabolas.

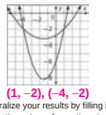
For 1–6, use the graph to estimate the solution(s) of the quadratic-quadratic system.

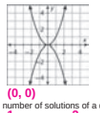
1. 
(−1, 0), (3, 0)

2. 
(−1, 0), (3, 0)

3. 
(−1, 0)

4. 
no solution

5. 
(1, −2), (−4, −2)

6. 
(0, 0)

7. Generalize your results by filling in the blanks: The number of solutions of a quadratic-quadratic system of equations is **0**, **1**, or **2**.

SECTION 8B

MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and skills in a real-world format.

INTERACTIVE **Online Edition**

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

Learn It Online
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CHAPTER 8

SECTION 8B

MULTI-STEP TEST PREP

MATHEMATICAL PRACTICES **Make sense of problems and persevere in solving them.**

Solving Quadratic Equations

Seeing Green A golf player hits a golf ball from a tee with an initial vertical velocity of 80 feet per second. The height of the golf ball t seconds after it is hit is given by $h = -16t^2 + 80t$.

1. How long is the golf ball in the air? **5 s**
2. What is the maximum height of the golf ball? **100 ft**
3. How long after the golf ball is hit does it reach its maximum height? **2.5 s**
4. What is the height of the golf ball after 3.5 seconds? **84 ft**
5. At what times is the golf ball 64 feet in the air? Explain.
At 1 s and 4 s; using the table function in a graphing calculator, X = 1 and 4 when $Y_1 = 64$.

(t) PhotoDisc/Getty Images; (b) ©COMSTOCK, Inc.; (tr) Stuart Franklin/Getty Images; (b) Robert Laberge/Getty Images

INTERVENTION

Scaffolding Questions

1. What is a reasonable domain for this situation? $0 \le t \le 5$
2. What is a reasonable range for this situation? $0 \le h \le 100$
3. What point represents the maximum height? the vertex
4. Does this problem ask for the value of an independent or dependent variable? dependent
5. What equation could you solve in order to answer this question? $64 = -16t^2 + 80t$

Extension

How would the function change if the golfer hit the ball with an initial velocity of 60 ft/s? How would the graph and the reasonable domain and range change? $h = -16t^2 + 60t$; the graph in the first quadrant would be shorter and not as wide; reasonable domain: $0 \le t \le 3.75$; reasonable range: $0 \le h \le 56.25$.

READY TO GO ON?

CHAPTER 8

SECTION 8B

Quiz for Lessons 8-5 Through 8-10

8-5 Solving Quadratic Equations by Graphing

Solve each equation by graphing the related function.

1. $x^2 - 9 = 0$ 3, −3
2. $x^2 + 3x - 4 = 0$ 1, −4
3. $4x^2 + 8x = 32$ 2, −4
4. The height of a fireworks rocket launched from a platform 35 meters above the ground can be approximated by $h = -5t^2 + 30t + 35$, where h is the height in meters and t is the time in seconds. Find the time it takes the rocket to reach the ground after it is launched. 7 s

8-6 Solving Quadratic Equations by Factoring

Use the Zero Product Property to solve each equation.

5. $(x + 1)(x + 3) = 0$ −1, −3
6. $(x - 6)(x - 3) = 0$ 6, 3
7. $(x + 6)(x - 3) = 0$ −6, 3
8. $(x + 7)(x - 10) = 0$ −7, 10

Solve each quadratic equation by factoring.

9. $x^2 - 4x - 32 = 0$ 9. 8, −4
10. $x^2 - 8x + 15 = 0$ 10. 3, 5
11. $x^2 + x = 6$ 2, −3
12. $-8x - 33 = -x^2$ 11, −3

8-7 Solving Quadratic Equations by Using Square Roots

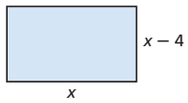
Solve using square roots.

13. $3x^2 = 48$ 4, −4
14. $36x^2 - 49 = 0$ $\frac{7}{6}, -\frac{7}{6}$
15. $-12 = x^2 - 21$ 3, −3

8-8 Completing the Square

Solve by completing the square.

16. $x^2 + 2x = 3$ 1, −3
17. $x^2 - 5 = 2x$ $1 \pm \sqrt{6}$
18. $x^2 + 7x = 8$ 1, −8
19. The width of a rectangle is 4 feet shorter than its length. The area of the rectangle is 42 square feet. Find the length and width. Round your answer to the nearest tenth of a foot. length: 8.8 ft; width: 4.8 ft



8-9 Using the Quadratic Formula and the Discriminant

Solve using the Quadratic Formula. Round your answer to the nearest hundredth.

20. $x^2 + 5x + 1 = 0$ −0.21, −4.79
21. $3x^2 + 1 = 2x$ no real solutions
22. $5x + 8 = 3x^2$ −1, 2.67

Find the number of real solutions of each equation using the discriminant.

23. $2x^2 - 3x + 4 = 0$ no real solutions
24. $x^2 + 1 + 2x = 0$ 1 real solution
25. $x^2 - 5 + 4x = 0$ 2 real solutions

8-10 Nonlinear Systems

26. Solve the system by graphing. Check your answer. $\begin{cases} y = x^2 - 7 \\ y = x - 7 \end{cases}$ (0, −7), (1, −6)
27. Solve the system by substitution. Check your answer. $\begin{cases} y = 2x^2 + x + 7 \\ y = 4x + 5 \end{cases}$ (0.5, 7), (2, 13)

READY TO GO ON?

SECTION 8B

Organizer

Objective: Assess students' mastery of concepts and skills in this section.

Resources

Assessment Resources
Section Quiz

INTERVENTION

Resources

***Ready to Go On? Intervention and Enrichment* Worksheets**

***Ready to Go On?* Online**

READY TO GO ON?

Diagnose and Prescribe

NO INTERVENE

YES ENRICH

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 8B: Worksheets	*Ready to Go On? Intervention*, Section 8B: Online	
Lesson 8-5	8-5 Intervention	Activity 8-5	Diagnose and Prescribe Online
Lesson 8-6	8-6 Intervention	Activity 8-6	
Lesson 8-7	8-7 Intervention	Activity 8-7	
Lesson 8-8	8-8 Intervention	Activity 8-8	
Lesson 8-9	8-9 Intervention	Activity 8-9	
Lesson 8-10	8-10 Intervention	Activity 8-10	

***Ready to Go On? Enrichment*, Section 8B**
Worksheets
Online

CHAPTER 8 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Recognize and graph cubic functions.

Solve cubic equations.

Online Edition

Countdown

Using the Extension

In this chapter, students learned to graph quadratic functions and solve quadratic equations. In this lesson, students apply similar techniques to graphing cubic functions and solving cubic equations.

COMMON CORE

CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
CC.9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve,….
Also **CC.9-12.A.REI.11*, CC.9-12.A.APR.3**

Learn It Online
State Resources Online

COMMON CORE

EXTENSION

Cubic Functions and Equations

CC.9-12.F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. *Also* **CC.9-12.A.REI.10, CC.9-12.A.REI.11*, CC.9-12.A.APR.3**

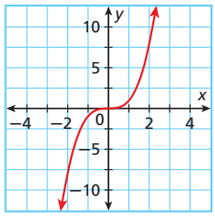
Objectives
Recognize and graph cubic functions.

Solve cubic equations.

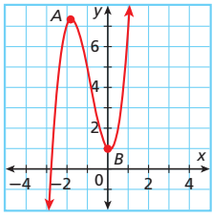
Vocabulary
cubic function
cubic equation

A **cubic function** is a function that can be written in the form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$. The parent cubic function is $f(x) = x^3$. To graph this function, choose several values of x and find ordered pairs.

x	$f(x)$
2	8
1	1
0	0
−1	−1
−2	−8



From the graph of $f(x) = x^3$, you can see
- the general shape of a cubic function.
- that the domain and the range are all real numbers.
- that the x-intercept and the y-intercept are both 0.



The graph of $f(x) = 2x^3 + 5x^2 - x + 1$ illustrates another characteristic of the graphs of cubic functions. Points A and B are called *turning points*. In general, the graph of a cubic function will have two turning points.

EXAMPLE 1 Graphing Cubic Functions

Graph $f(x) = -2x^3 + 3x^2 + x - 4$. Identify the intercepts and give the domain and range.

x	$f(x) = -2x^3 + 3x^2 + x - 4$	$f(x)$
−1	$-2(-1)^3 + 3(-1)^2 - 1 - 4$	0
0	$-2(0)^3 + 3(0)^2 + 0 - 4$	−4
1	$-2(1)^3 + 3(1)^2 + 1 - 4$	−2
2	$-2(2)^3 + 3(2)^2 + 2 - 4$	−6

Choose positive, negative, and zero values for x, and find ordered pairs.

1 Introduce

Motivate

Show students the formulas for the volume of a sphere and a cube.

cube: $V = s^3$

sphere: $V = \frac{4}{3}\pi r^3$

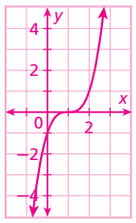
Help students recognize that both of these are cubic functions.

2 Teach

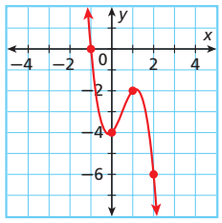
Guided Instruction

There are three basic strategies for solving cubic equations. One strategy is to graph the related function. The x-intercepts of the function are the solutions of the equation. A second strategy is to rewrite the equation so that one side is equal to 0, factor the cubic polynomial into a linear factor and a quadratic factor, and apply the techniques for solving linear and quadratic equations. A third strategy is to rewrite the equation in the form *cubed expression* = *constant* and to take the cube root of both sides.

1a.



x-int.: 1; *y*-int.: –1;
D: all real numbers;
R: all real numbers

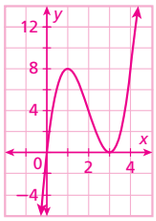


Plot the ordered pairs and connect them with a smooth curve.

Notice that, in general, this graph falls from left to right. This is because the value of a is negative.

The *x*-intercept is –1. The *y*-intercept is –4. The domain and range are all real numbers.

1b.



x-int.: 0, 3; *y*-int.: 0;
D: all real numbers;
R: all real numbers

CHECK IT OUT! **Graph each cubic function. Identify the intercepts and give the domain and range.**

1a. $f(x) = (x - 1)^3$ **1b.** $f(x) = 2x^3 - 12x^2 + 18x$

Previously, you saw that every quadratic function has a related quadratic equation. Cubic functions also have related *cubic equations*. A **cubic equation** is an equation that can be written in the form $ax^3 + bx^2 + cx + d = 0$, where $a \neq 0$.

One way to solve a cubic equation is by graphing the related function and finding its zeros.

EXAMPLE 2 Solving Cubic Equations by Graphing

Solve $x^3 - 2x^2 - x = -2$ by graphing. Check your answer.

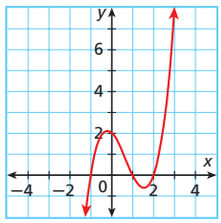
Step 1 Rewrite the equation in the form $ax^3 + bx^2 + cx + d = 0$.

$x^3 - 2x^2 - x = -2$

$x^3 - 2x^2 - x + 2 = 0$ *Add 2 to both sides of the equation.*

Step 2 Write and graph the related function: $f(x) = x^3 - 2x^2 - x + 2$

x	$f(x) = x^3 - 2x^2 - x + 2$	$f(x)$
–1	$(-1)^3 - 2(-1)^2 - (-1) + 2$	0
0	$(0)^3 - 2(0)^2 - 0 + 2$	2
1	$(1)^3 - 2(1)^2 - 1 + 2$	0
2	$(2)^3 - 2(2)^2 - 2 + 2$	0
3	$(3)^3 - 2(3)^2 - 3 + 2$	8



Step 3 Find the zeros.

The zeros appear to be –1, 1, and 2. Check these values in the original equation.

$x^3 - 2x^2 - x = -2$	
$(-1)^3 - 2(-1)^2 - (-1)$	–2
$(-1) - 2 + 1$	–2
–2	–2 ✓

$x^3 - 2x^2 - x = -2$	
$1^3 - 2(1)^2 - 1$	–2
$1 - 2 - 1$	–2
–2	–2 ✓

$x^3 - 2x^2 - x = -2$	
$2^3 - 2(2)^2 - 2$	–2
$x - 8 - 2$	–2
–2	–2 ✓

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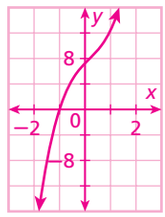
Additional Examples

Example 1

Graph $f(x) = 2x^3 - x^2 + 4x + 7$. Identify the intercepts and give the domain and range.



x-int.: −1; *y*-int.: 7; D: all real numbers; R: all real numbers

Example 2

Solve each equation by graphing. Check your answer.

A. $x^3 - 13x + 12 = 0$ −4, 1, 3

B. $3x^3 - 15x^2 - 12x = -60$ −2, 2, 5

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- What is the shape of the graph of a cubic function?

EXAMPLE 2

- Why must you write the cubic equation so that one side is equal to 0?
- Can a cubic equation have no real solutions?

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Additional Examples

Example 3

Solve each equation. Check your answer.

A. $4x^3 - 5x^2 + 2x = 3$

$0, \frac{5 \pm \sqrt{57}}{8}$

B. $5x^3 = 20x$ 0, 2, −2

C. $(2x + 1)^3 = -125$ −3

Also available online

Questioning Strategies

EXAMPLE 3

- When can you solve a cubic equation by taking the cube root of both sides of the equation?

CHECK IT OUT! **Solve each equation by graphing. Check your answer.**

2a. $x^3 - 2x^2 - 25x = -50$ **–5, 2, 5** **2b.** $2x^3 + 12x^2 = 30x + 200$ **–5, 4**

Cubic equations can also be solved algebraically. Many of the methods used to solve quadratic equations can be applied to cubic equations as well.

EXAMPLE 3 Solving Cubic Equations Algebraically

Solve each equation. Check your answer.

A $(x + 5)^3 = 27$

$\sqrt[3]{(x+5)^3} = \sqrt[3]{27}$ *Take the cube root of both sides.*

$x + 5 = 3$

$x = -2$ *Subtract 5 from both sides.*

Check

$(x+5)^3 = 27$	
$(-2+5)^3$	27
3^3	27
27	27 ✓

Substitute –2 for x in the original equation.

B $x^3 + 3x^2 = -2x$

$x^3 + 3x^2 + 2x = 0$ *Add 2x to both sides.*

$x(x^2 + 3x + 2) = 0$ *Factor out x on the left side.*

$x(x + 1)(x + 2) = 0$ *Factor the quadratic trinomial.*

$x = 0$ or $x + 1 = 0$ or $x + 2 = 0$ *Zero Product Property*

$x = -1$ or $x = -2$ *Solve each equation.*

The solutions are 0, –1, and –2.

Remember!

The factored expression must equal zero to use the Zero Product Property.

Check

$x^3 + 3x^2 = -2x$	
$0^3 + 3(0)^2$	$-2(0)$
$0 + 0$	0
0	0 ✓

$x^3 + 3x^2 = -2x$	
$(-1)^3 + 3(-1)^2$	$-2(-1)$
$-1 + 3(1)$	2
$-1 + 3$	2
2	2 ✓

$x^3 + 3x^2 = -2x$	
$(-2)^3 + 3(-2)^2$	$-2(-2)$
$-8 + 3(4)$	4
$-8 + 12$	4
4	4 ✓

C $x^3 - 3.125x = -1.25x^2$

$x^3 + 1.25x^2 - 3.125x = 0$ *Add $1.25x^2$ to both sides.*

$x(x^2 + 1.25x - 3.125) = 0$ *Factor out x on the left side.*

$x = 0$ or $x^2 + 1.25x - 3.125 = 0$ *Zero Product Property*

$x = \frac{-1.25 \pm \sqrt{(1.25)^2 - 4(1)(-3.125)}}{2(1)}$ *Quadratic Formula*

$x = \frac{-1.25 \pm 3.75}{2}$ *Simplify.*

$x = -2.5$ or $x = 1.25$

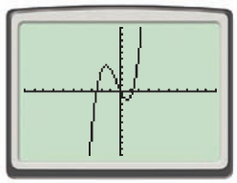
The solutions are –2.5, 0, and 1.25.

3 Close

Summarize

Ask one group of students to solve $2x^3 - 14x^2 + 24x = 0$ by graphing. Ask another group to solve the same equation algebraically. Have the groups present their solutions. 0, 3, 4

Check Use a graphing calculator.



Graph the related function and look for the zeros.

The solutions look reasonable.

3b. $0, \frac{3+\sqrt{5}}{2} \approx 2.62, \frac{3-\sqrt{5}}{2} \approx 0.38$

CHECK IT OUT!

Solve each equation. Check your answer.

3a. $(x+2)^3 = 64$ 2 **3b.** $4x^3 - 12x^2 + 4x = 0$ **3c.** $x^3 + 3x^2 = 10x$ –5, 0, 2

EXTENSION Exercises

Graph each cubic function. Identify the intercepts and give the domain and range.

1. $f(x) = x^3 - 2x^2 + 3x + 6$ **2.** $g(x) = -4x^3 + 2x - 2$

Solve each equation by graphing. Check your answer.

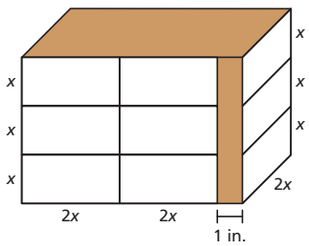
3. $2x^3 - 6x = -4x^2$ –3, 0, 1 **4.** $-3x^3 + 12x^2 + 12x = 48$ –2, 2, 4

Solve each equation. Check your answer.

7. $0, \frac{3+\sqrt{89}}{2} \approx 6.22, \frac{3-\sqrt{89}}{2} \approx -3.22$

5. $(x-9)^3 = 64$ 13 **6.** $8x + 4x^2 = 4x^3$ 2, –1, 0 **7.** $5x^3 + 3x^2 = 4x$

8. The Send-It Store uses shipping labels that are x in. tall and $2x$ in. wide. Six labels fit on the front of the store's standard shipping box with an area of $3x$ in^2 left over. Three labels fit on the side of the box. The volume of the box is $108x$ in^3. What is the area of one label? 8 in^2



9. a. Graph the functions $f(x) = x^3$, $f(x) = x^3 + 1$, and $f(x) = x^3 + 2$ on the same coordinate plane. Describe any patterns you observe. Predict the shape of the graph of $f(x) = x^3 + c$.

b. Graph the functions $g(x) = x^3$, $g(x) = (x-1)^3$, and $g(x) = (x-2)^3$ on the same coordinate plane. Describe any patterns you observe. Predict the shape of the graph of $g(x) = (x-c)^3$.

Use a graphing calculator to find the approximate solution(s) of each cubic equation. Round to the nearest hundredth.

14. 1, 2, or 3; a cubic equation will have 1, 2, or 3 real solutions.

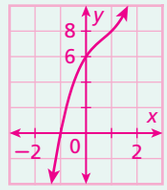
10. $100x^3 - 40x^2 = 6x$ –0.12, 0, 0.52 **11.** $2x^3 - 5x^2 + 4x = -3$ –0.45

12. $-1.32x^3 - 3.65x^2 = -0.43x$ –2.88, 0, 0.11 **13.** $\frac{3}{5}x^3 + x^2 - \frac{1}{2}x = 9$ 2.12

14. Critical Thinking How many zeros can a cubic function have? What does this tell you about the number of real solutions possible for a cubic equation?

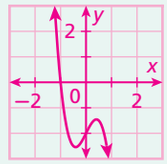
Answers

1.



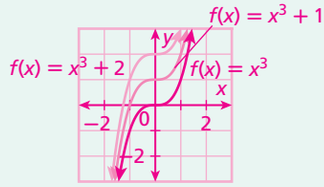
x-int.: –1; *y*-int.: 6; D: all real numbers; R: all real numbers

2.



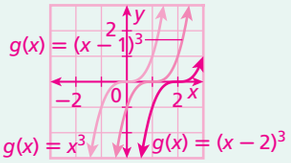
x-int.: –1; *y*-int.: –2; D: all real numbers; R: all real numbers

9a.



The graphs are the same shape, but each is a vertical translation of the others. The graph of $f(x) = x^3 + c$ will be the graph of $f(x) = x^3$ translated c units vertically.

b.



The graphs are the same shape, but each is a horizontal translation of the others. The graph of $g(x) = (x-c)^3$ will be the graph of $g(x) = x^3$ translated c units horizontally.

Organizer

Objective: Help students organize and review key concepts and skills presented in this chapter.

Online Edition
Multilingual Glossary

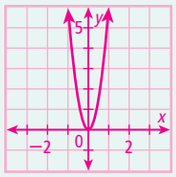
Countdown

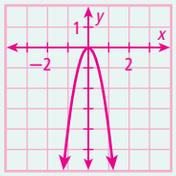
Resources

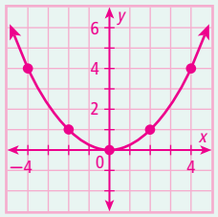
Learn It Online
Multilingual Glossary

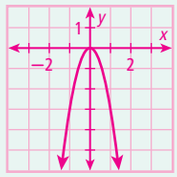
Lesson Tutorial Videos

Answers

1. vertex
2. zero of a function
3. Yes; it is in standard form.
4. No; $a = 0$
5. Yes; it is in standard form.
6. No; a quadratic function does not have a power of x greater than 2.
7. 

8. 

9. 

10. 

11. Upward; the value of a is positive.
12. Downward; the value of a is negative.
13. $-5, 2$
14. $-1, 2$

Vocabulary

axis of symmetry	**minimum**	**quadratic function**
completing the square	**nonlinear system of equations**	**vertex**
discriminant	**parabola**	**zero of a function**
maximum	**quadratic equation**	

Complete the sentences below with vocabulary words from the list above.

1. The ___?___ is the highest or lowest point on a parabola.
2. A ___?___ can also be called an x-intercept of the function.

8-1 Identifying Quadratic Functions

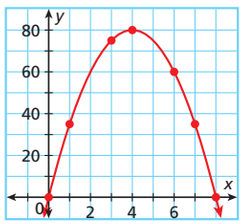
EXAMPLE

■ **Use a table of values to graph $y = -5x^2 + 40x$.**

Step 1 Make a table of values. Choose values of x and use them to find values of y.

x	0	1	3	4	6	7	8
y	0	35	75	80	60	35	0

Step 2 Plot the points and connect them.



EXERCISES

Tell whether each function is quadratic. Explain.

3. $y = 2x^2 + 9x - 5$
4. $y = -4x + 3$
5. $y = -\frac{1}{2}x^2$
6. $y = 5x^3 + 8$

Use a table of values to graph each quadratic function.

7. $y = 6x^2$
8. $y = -4x^2$
9. $y = \frac{1}{4}x^2$
10. $y = -3x^2$

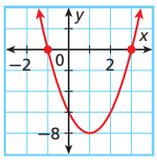
Tell whether the graph of each function opens upward or downward. Explain.

11. $y = 5x^2 - 12$
12. $y = -x^2 + 3x - 7$

8-2 Characteristics of Quadratic Functions

EXAMPLE

■ **Find the zeros of $y = 2x^2 - 4x - 6$ from its graph.**

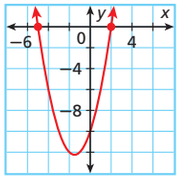


Use the graph to find the zeros.
The zeros are -1 and 3.

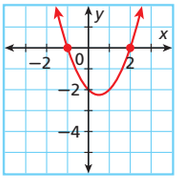
EXERCISES

Find the zeros of each quadratic function from its graph. Check your answer.

13. $y = x^2 + 3x - 10$



14. $y = x^2 - x - 2$



8-3 Graphing Quadratic Functions

EXAMPLE

■ **Graph $y = 2x^2 - 8x - 10$.**

Step 1 Find the axis of symmetry.

$$x = \frac{-b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$$

The axis of symmetry is $x = 2$.

Step 2 Find the vertex.

$y = 2x^2 - 8x - 10$
$y = 2(2)^2 - 8(2) - 10$
$y = -18$

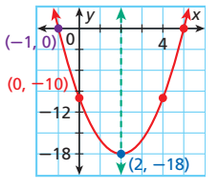
The vertex is $(2, -18)$.

Step 3 Find the y-intercept.
$c = -10$

Step 4 Find one more point on the graph.

$y = 2(-1)^2 - 8(-1) - 10 = 0$ *Let $x = -1$.*
Use $(-1, 0)$.

Step 5 Graph the axis of symmetry and the points. **Reflect** the points and connect with a smooth curve.



EXERCISES

Graph each quadratic function.

15. $y = x^2 + 6x + 6$

16. $y = x^2 - 4x - 12$

17. $y = x^2 - 8x + 7$

18. $y = 2x^2 - 6x - 8$

19. $3x^2 + 6x = y - 3$

20. $2 - 4x^2 + y = 8x - 10$

21. Water that is sprayed upward from a sprinkler with an initial velocity of 20 m/s can be approximated by the function $y = -5x^2 + 20x$, where y is the height of a drop of water x seconds after it is released. Graph this function. Find the time it takes a drop of water to reach its maximum height, the water's maximum height, and the time it takes the water to reach the ground.

8-4 Transforming Quadratic Functions

EXAMPLE

■ **Compare the graph of $g(x) = 3x^2 - 4$ with the graph of $f(x) = x^2$. Use the functions.**

- Both graphs open upward because $a > 0$.
- The axis of symmetry is the same, $x = 0$, because $b = 0$ in both functions.
- The graph of $g(x)$ is narrower than the graph of $f(x)$ because $|3| > |1|$.
- The vertex of $f(x)$ is $(0, 0)$. The vertex of $g(x)$ is translated 4 units down to $(0, -4)$.
- $f(x)$ has one zero at the origin. $g(x)$ has two zeros because the vertex is below the origin and the parabola opens upward.

EXERCISES

Compare the widths of the graphs of the given quadratic functions. Order functions with different widths from narrowest graph to widest.

22. $f(x) = 2x^2$, $g(x) = 4x^2$

23. $f(x) = 6x^2$, $g(x) = -6x^2$

24. $f(x) = x^2$, $g(x) = \frac{1}{3}x^2$, $h(x) = 3x^2$

Compare the graph of each function with the graph of $f(x) = x^2$.

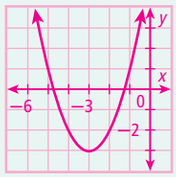
25. $g(x) = x^2 + 5$

26. $g(x) = 3x^2 - 1$

27. $g(x) = 2x^2 + 3$

Answers

15.



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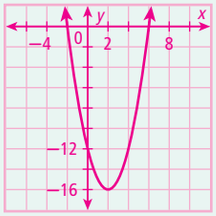

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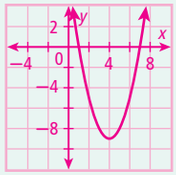

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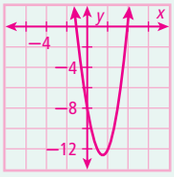

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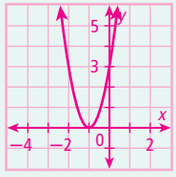

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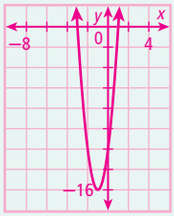

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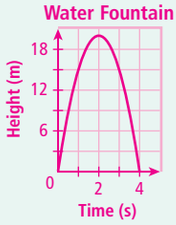


In 2 seconds, the water reaches its maximum height of 20 meters. It takes a total of 4 seconds for the water to reach the ground.

22. $g(x)$, $f(x)$

23. The graphs have the same width.

24. $h(x)$, $f(x)$, $g(x)$

25. The graph of $g(x)$ has the same width, has the same axis of symmetry, also opens upward, and its vertex is translated 5 units up to $(0, 5)$.

26. The graph of $g(x)$ has the same axis of symmetry, and also opens upward, but it is narrower, and its vertex is translated 1 unit down to $(0, -1)$.

27. The graph of $g(x)$ is narrower, has the same axis of symmetry, also opens upward, and its vertex is translated 3 units up to $(0, 3)$.

Answers

28. $x = -3$ or $x = -1$
29. $x = -3$
30. no real solutions
31. $x = 1$ or $x = 5$
32. $x = 4$
33. $x = \pm 1$
34. no real solutions
35. $x = -5$ or $x = -1$
36. $x = -7$ or $x = -2$
37. $x = -3$ or $x = 5$
38. $x = -1$ or $x = 2$
39. $x = -5$
40. $x = 4.5$
41. $x^2 + 2x = 48$; 6 ft
42. $x = \pm 8$
43. $x = \pm 12$
44. no real solutions
45. $x = 0$
46. $x = \pm 5$
47. $x = \pm \frac{5}{2}$
48. 4 ft

8-5 Solving Quadratic Equations by Graphing

EXAMPLE

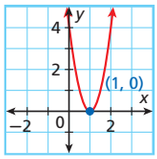
■ **Solve $-4 = 4x^2 - 8x$ by graphing the related function.**

Step 1 Write the equation in standard form.

$0 = 4x^2 - 8x + 4$

Step 2 Graph the related function.

$y = 4x^2 - 8x + 4$



Step 3 Find the zeros.
The only zero is 1. The solution is $x = 1$.

EXERCISES

Solve each equation by graphing the related function.

28. $0 = x^2 + 4x + 3$
29. $0 = x^2 + 6x + 9$
30. $-4x^2 = 3$
31. $x^2 + 5 = 6x$
32. $-4x^2 = 64 - 32x$
33. $9 = 9x^2$
34. $-3x^2 + 2x = 5$

8-6 Solving Quadratic Equations by Factoring

EXAMPLE

■ **Solve $3x^2 - 6x = 24$ by factoring.**

$3x^2 - 6x = 24$ *Write the equation in standard form.*
$3x^2 - 6x - 24 = 0$
$3(x^2 - 2x - 8) = 0$ *Factor out 3.*
$3(x + 2)(x - 4) = 0$ *Factor the trinomial.*
$3 \neq 0$, $x + 2 = 0$ or $x - 4 = 0$ *Zero Product Property*
$x = -2$ or $x = 4$ *Solve each equation.*

EXERCISES

Solve each quadratic equation by factoring.

35. $x^2 + 6x + 5 = 0$
36. $x^2 + 9x + 14 = 0$
37. $x^2 - 2x - 15 = 0$
38. $2x^2 - 2x - 4 = 0$
39. $x^2 + 10x + 25 = 0$
40. $4x^2 - 36x = -81$
41. A rectangle is 2 feet longer than it is wide. The area of the rectangle is 48 square feet. Write and solve an equation that can be used to find the width of the rectangle.

8-7 Solving Quadratic Equations by Using Square Roots

EXAMPLE

■ **Solve $2x^2 = 98$ using square roots.**

$\frac{2x^2}{2} = \frac{98}{2}$ *Divide both sides of the equation by 2 to isolate x^2.*
$x^2 = 49$
$x = \pm\sqrt{49}$ *Take the square root of both sides.*
$x = \pm 7$ *Use $\pm$ to show both roots.*

The solutions are -7 and 7.

EXERCISES

Solve using square roots.

42. $5x^2 = 320$
43. $-x^2 + 144 = 0$
44. $x^2 = -16$
45. $x^2 + 7 = 7$
46. $2x^2 = 50$
47. $4x^2 = 25$
48. A rectangle is twice as long as it is wide. The area of the rectangle is 32 square feet. Find the rectangle's width.

8-8 Completing the Square

EXAMPLE

■ **Solve $x^2 - 6x = -5$ by completing the square.**

$\left(\frac{-6}{2}\right)^2 = 9$ *Find $\left(\frac{b}{2}\right)^2$.*

$x^2 - 6x + 9 = -5 + 9$

$x^2 - 6x + 9 = 4$ *Complete the square.*

$(x - 3)^2 = 4$ *Factor the trinomial.*

$x - 3 = \pm\sqrt{4}$ *Take the square root of both sides.*

$x - 3 = \pm 2$ *Use the ± symbol.*

$x - 3 = 2$ or $x - 3 = -2$ *Solve each equation.*

$x = 5$ or $x = 1$

The solutions are 5 and 1.

EXERCISES

Solve by completing the square.

49. $x^2 + 2x = 48$

50. $x^2 + 4x = 21$

51. $2x^2 - 12x + 10 = 0$

52. $x^2 - 10x = -20$

53. A homeowner is planning an addition to her house. She wants the new family room to be a rectangle with an area of 192 square feet. The contractor says that the length needs to be 4 more feet than the width. What will the dimensions of the new room be?

8-9 The Quadratic Formula and the Discriminant

EXAMPLE

■ **Solve $x^2 + 4x + 4 = 0$ using the Quadratic Formula.**

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *Write the Quadratic Formula.*

$= \frac{-4 \pm \sqrt{4^2 - 4(1)(4)}}{2(1)}$ *Substitute for a, b, and c.*

$= \frac{-4 \pm \sqrt{16 - 16}}{2}$ *Simplify.*

$= \frac{-4 \pm \sqrt{0}}{2} = \frac{-4}{2} = -2$

The solution is $x = -2$.

EXERCISES

Solve using the Quadratic Formula.

54. $x^2 - 5x - 6 = 0$

55. $2x^2 - 9x - 5 = 0$

56. $4x^2 - 8x + 4 = 0$

57. $x^2 - 6x = -7$

Find the number of real solutions of each equation using the discriminant.

58. $x^2 - 12x + 36 = 0$

59. $3x^2 + 5 = 0$

60. $2x^2 - 13x = -20$

61. $6x^2 - 20 = 15x + 1$

8-10 Nonlinear Systems

EXAMPLE

■ **Solve $\begin{cases} y = x^2 \\ y = x + 2 \end{cases}$ by substitution.**

$x + 2 = x^2$. *Substitute $x + 2$ for y in the first equation.*

$0 = x^2 - x - 2$

$0 = (x - 2)(x + 1)$ *Factor the trinomial.*

$x - 2 = 0$ or $x + 1 = 0$ *Solve the equations.*

$x = 2$ or $x = -1$

$y = x + 2$	$y = x + 2$	*Write one of the original equations.*
$y = 2 + 2$	$y = -1 + 2$	*Substitute each x-value and solve for y.*
$y = 4$	$y = 1$	

The solutions are (2, 4) and (−1, 1).

EXERCISES

Solve each system.

62. $\begin{cases} y = x^2 - 4 \\ y = -2x - 1 \end{cases}$

63. $\begin{cases} y = x^2 - 3x - 1 \\ y = 2x - 5 \end{cases}$

64. $\begin{cases} y = x^2 + 5x + 4 \\ y = -2x - 6 \end{cases}$

Answers

49. $x = -8$ or $x = 6$

50. $x = -7$ or $x = 3$

51. $x = 1$ or $x = 5$

52. $x = 5 \pm \sqrt{5}$

53. 16 ft by 12 ft

54. $x = -1$ or $x = 6$

55. $x = -\frac{1}{2}$ or $x = 5$

56. $x = 1$

57. $x = \frac{6 \pm \sqrt{8}}{2}$

58. 1

59. 0

60. 2

61. 2

62. (−3, 5), (1, −3)

63. (1, −3), (4, 3)

64. (−2, −2), (−5, 4)

Study Guide: Review **607**

CHAPTER 8

CHAPTER TEST

Organizer

Objective: Assess students' mastery of concepts and skills in this chapter.

INTERACTIVE **Online Edition**

Resources

Assessment Resources

Chapter Tests

- Free Response (Levels A, B, C)
- Multiple Choice (Levels A, B, C)
- Performance Assessment

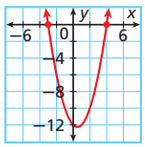
Learn It Online
State Resources Online

CHAPTER 8

CHAPTER TEST

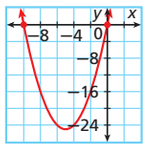
Tell whether each function is quadratic. Explain.

1. $\{(10, 50), (11, 71), (12, 94), (13, 119), (14, 146)\}$
2. $3x^2 + y = 4 + 3x^2$ **No; $a = 0$**
3. Tell whether the graph of $y = -2x^2 + 7x - 5$ opens upward or downward and whether the parabola has a maximum or a minimum. **downward; maximum**
4. Estimate the zeros of the quadratic function.



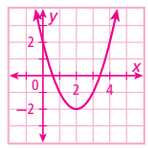
−3 and 4

5. Find the axis of symmetry of the parabola.



$x = -5$

6. Find the vertex of the graph of $y = x^2 + 6x + 8$. **$(-3, -1)$**
7. Graph the quadratic function $y = x^2 - 4x + 2$.



Compare the graph of each function with the graph of $f(x) = x^2$.

8. $g(x) = -x^2 - 2$
9. $h(x) = \frac{1}{3}x^2 + 1$
10. $g(x) = 3x^2 - 4$
11. A hammer is dropped from a 40-foot scaffold. Another one is dropped from a 60-foot scaffold.
    a. Write the two height functions and compare their graphs. Use $h(t) = -16t^2 + c$, where c is the height of the scaffold.
    b. Use the graphs to estimate when each hammer will reach the ground. **about 1.6 s and about 1.9 s**
12. A rocket is launched with an initial vertical velocity of 110 m/s. The height of the rocket in meters is approximated by the quadratic equation $h = -5t^2 + 110t$ where t is the time after launch in seconds. About how long does it take for the rocket to return to the ground? **about 22 s**

Solve each quadratic equation by factoring.

13. $x^2 + 6x + 5 = 0$ **−5, −1**
14. $x^2 - 12x = -36$ **6**
15. $x^2 - 81 = 0$ **−9, 9**

Solve by using square roots.

16. $-2x^2 = -72$ **± 6**
17. $9x^2 - 49 = 0$ **$\pm \frac{7}{3}$**
18. $3x^2 + 12 = 0$ **no real solutions**

Solve by completing the square.

19. $x^2 + 10x = -21$ **−7, −3**
20. $x^2 - 6x + 4 = 0$ **$3 \pm \sqrt{5}$**
21. $2x^2 + 16x = 0$ **−8, 0**

Solve each quadratic equation. Round to the nearest hundredth if necessary.

22. $x^2 + 3x - 40 = 0$ **−8, 5**
23. $2x^2 + 7x = -5$ **$-\frac{5}{2}$, −1**
24. $8x^2 + 3x - 1 = 0$ **$\approx -0.59, \approx 0.21$**

Find the number of real solutions of each equation using the discriminant.

25. $4x^2 - 4x + 1 = 0$ **1**
26. $2x^2 + 5x - 25 = 0$ **2**
27. $\frac{1}{2}x^2 + 8 = 0$ **no real solutions**
28. Solve the system. $\begin{cases} y = x^2 + 7x + 6 \\ y = x + 1 \end{cases}$ **$(-1, 0), (-5, -4)$**

Answers

1. Yes; the second differences of the y-coordinates are constant.
8. The graph of $g(x)$ is the same width, has the same axis of symmetry, opens downward, and its vertex is translated 2 units down to $(0, -2)$.
9. The graph of $h(x)$ is wider, has the same axis of symmetry, opens upward, and its vertex is translated 1 unit up.
10. The graph of $g(x)$ is narrower, has the same axis of symmetry, opens upward, and its vertex is translated 4 units down.

11a. $h_1(t) = -16t^2 + 40$; $h_2(t) = -16t^2 + 60$; The graph of h_2 is a vertical translation of h_1. The y-intercept of the graph of h_2 is 20 units higher.

COLLEGE ENTRANCE EXAM PRACTICE

CHAPTER 8

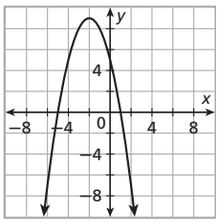
FOCUS ON SAT SUBJECT TESTS

In addition to the SAT, some colleges require the SAT Subject Tests for admission. Colleges that don't require the SAT Subject Tests may still use the scores to learn about your academic background and to place you in the appropriate college math class.

HOT TIP! Take the SAT Subject Test in mathematics while the material is still fresh in your mind. You are not expected to be familiar with all of the test content, but you should have completed at least three years of college-prep math.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

1. The graph below corresponds to which of the following quadratic functions?



(A) $f(x) = x^2 + 4x - 5$

(B) $f(x) = -x^2 - 4x + 3$

(C) $f(x) = -x^2 + 5x - 4$

(D) $f(x) = -x^2 - 4x + 5$

(E) $f(x) = -x^2 - 3x + 5$

2. What is the sum of the solutions to the equation $9x^2 - 6x = 8$?

(A) $\frac{4}{3}$

(B) $\frac{2}{3}$

(C) $\frac{1}{3}$

(D) $-\frac{2}{3}$

(E) $-\frac{8}{3}$

3. If $h(x) = ax^2 + bx + c$, where $b^2 - 4ac < 0$ and $a < 0$, which of the following statements must be true?

I. The graph of $h(x)$ has no points in the first or second quadrants.

II. The graph of $h(x)$ has no points in the third or fourth quadrants.

III. The graph of $h(x)$ has points in all quadrants.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) None of the statements are true.

4. What is the axis of symmetry for the graph of a quadratic function whose zeros are -2 and 4?

(A) $x = -2$

(B) $x = 0$

(C) $x = 1$

(D) $x = 2$

(E) $x = 6$

5. How many real-number solutions does $0 = x^2 - 7x + 1$ have?

(A) None

(B) One

(C) Two

(D) All real numbers

(E) It is impossible to determine.

Organizer

Objective: Provide practice for college entrance exams such as the SAT Mathematics Subject Tests.

Online Edition

Resources

College Entrance Exam Practice

Questions on the SAT Mathematics Subject Tests Levels 1 and 2 represent the following math content areas:

	Level	
	1	2
Number and Operations	10–14%	10–14%
Algebra and Functions	38–42%	48–52%
Geometry and Measurement	38–42%	28–32%
Plane Euclidean	18–22%	0%
Coordinate	8–12%	10–14%
Three-dimensional	4–6%	4–6%
Trigonometry	6–8%	12–16%
Data Analysis, Statistics, and Probability	6–10%	6–10%

Items on this page focus on:

- Algebra and Functions

TEST PREP DOCTOR

1. Students who chose **B** found the correct axis of symmetry but not the correct vertex or y-intercept. Students who chose **E** found the correct y-intercept but did not find the correct axis of symmetry.

2. Students who chose **A** or **D** found just one of the solutions of the equation. Students who chose **E** may have tried to find the product of the solutions of the equation.

3. Students who chose **B** did not use the fact that $a < 0$. Students who chose **D** may not realize that if statements I and II are both true, then the graph of $h(x)$ is restricted to the axes.

4. Students who chose **E** found the distance between $x = -2$ and $x = 4$, not the vertical line halfway between them. Students who chose **A** might not understand the meaning of the axis of symmetry.

5. Students who chose **E** did not realize that the graph or the discriminant could be used to determine the answer. Students who chose **A** or **B** may have made a computational error in finding the discriminant or may not have applied it correctly.

CHAPTER 8 TEST TACKLER

Organizer

Objective: Provide opportunities to learn and practice common test-taking strategies.

INTERACTIVE **Online Edition**

TEST PREP DOCTOR This Test Tackler explains how students should write an explanation to an extended-response test item. Students need to be aware that even if their calculations are correct, the item will not receive full credit if their explanation does not clearly support their work.

Give students a problem from the chapter. Once students work through the problem, have them explain out loud to a partner how they determined their answer. When students are comfortable doing this, have them write it down in paragraph form. Guide students to use complete sentences.

As students rewrite the response in **Problem 5,** encourage them to write neatly. If students are struggling with forming complete sentences, have them list their thought process in bulleted form.

Extended Response: Explain Your Reasoning

Extended response test items often include multipart questions that evaluate your understanding of a math concept. To receive full credit, you must answer the problem correctly, show all of your work, and explain your reasoning. Use complete sentences and show your problem-solving method clearly.

EXAMPLE 1

Extended Response **Given $\frac{1}{2}x^2 + y = 4x - 3$ and $y = 2x - 12x$, identify which is a quadratic function. Provide an explanation for your decision. For the quadratic function, tell whether the graph of the function opens upward or downward and whether the parabola has a maximum or a minimum. Explain your reasoning.**

Read the solutions provided by two different students.

Student A

The quadratic function is $\frac{1}{2}x^2 + y = 4x - 3$ because it can be written in standard form, $y = -\frac{1}{2}x^2 + 4x - 3$, where a, b, and c are real numbers and $a \neq 0$. The other function, $y = 2x - 12x$, is not quadratic because there is no x^2- term.

The graph of this function will open downward because a, which is equal to $-\frac{1}{2}$, is less than 0. Because the parabola opens downward, the graph will have a maximum.

Excellent explanation

The response includes the correct answers along with a detailed explanation for each part of the problem. The explanation is written using complete sentences and is presented in an order that is easy to follow and to understand. It is obvious that this student knows how to determine and interpret a quadratic function.

Student B

$\frac{1}{2}x^2 + y = 4x - 3$ There is an x^2.

When I graphed the function on my calculator, I saw a parabola that opened downward.

It had a maximum.

Poor explanation

The response includes the correct answers, but the explanation does not include details. The reason for defining the function as quadratic does not show knowledge of the concept. The student shows a lack of understanding of how to write and interpret a quadratic function in standard form.

HOT TIP! Include as many details as possible to support your reasoning. This increases the chance of getting full credit for your response.

Read each test item and answer the questions that follow.

Item A

The height y in feet of a tennis ball x seconds after it is ejected from a serving machine is given by the ordered pairs $\{(0, 10), (0.5, 9), (1, 7), (1.5, 4), (2, 0)\}$. Determine whether the function is quadratic. Find its domain and range. Explain your answers.

1. What should a student include in the explanation to receive full credit?
2. Read the two explanations below. Which explanation is better? Why?

Student A

Range: $0 \le y \le 10$ Domain: $0 \le x \le 2$
Second differences are -1: quadratic

Student B

The function is quadratic because the second differences are constant: -1. The domain and range are determined by the points $(0, 10)$ and $(2, 0)$. The range is $0 \le y \le 10$, and the domain is $0 \le x \le 2$.

Item B

The height of a golf ball can be approximated by the function $y = -5x^2 + 20x + 8$, where y is the height in meters above the ground and x is the time in seconds after the ball is hit. What is the maximum height of the ball? How long does it take for the ball to reach its maximum height? Explain.

3. A student correctly found the following answers. Use this information to write a clear and concise explanation.

Axis of symmetry is the vertical line at $x = 2$.
Vertex is at $(2, 28)$.
28 meters; 2 seconds

Item C

A science teacher set off a bottle rocket as part of a lab experiment. The function $h = -16t^2 + 96t$ represents the height in feet of a rocket that is shot out of a bottle with an initial vertical velocity of 96 feet per second. Find the time that the rocket is in the air. Explain how you found your answer.

4. Read the two responses below.
   a. Which student provided the better explanation? Why?
   b. What advice would you give the other student to improve his or her explanation?

Student C

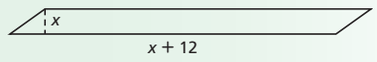
Graph the function $h = -16t^2 + 96t$, and then find the zeros. The first zero is when $t = 0$, when the rocket is launched. The second zero is when the rocket hits the ground: $t = 6$. The difference between 6 and 0 is the time that the rocket is in the air: 6 seconds.

Student D

6 seconds.
Graph the function to find how long the rocket is in the air, and find the values where it crosses the x-axis.

Item D

The base of a parallelogram is 12 centimeters more than its height. The area of the parallelogram is 13 square centimeters. Explain how to determine the height and base of the figure. What is the height? What is the base?



5. Read the following response. Identify any areas that need improvement. Rewrite the response so that it will receive full credit.

$x^2 + 12x + 36 = 49$ Complete the square.
$(x+6)^2 = 49$
$x + 6 = \sqrt{49}$
$x + 6 = \pm 7$; $x = 1$ or -13

base = 13, height = 1

Answers

Possible answers:

1. A student should include whether the function is quadratic, a table supporting the claim with 1st and 2nd differences in the y-values, the values of the domain and range, and an explanation of what the domain and range represent.
2. Student B gave the better explanation. It is written in complete sentences, and the thought process is clear. All parts of the question are explained thoroughly. Student A's explanation is out of order, it is not in complete sentences, and there is not enough support for the answers.
3. To find the axis of symmetry, substitute -5 for a and 20 for b in the formula $x = -\frac{b}{2a}$.
   $x = -\frac{20}{2(-5)}$
   $x = 2$
   To find the y-coordinate of the vertex, find the value of y when $x = 2$.
   $y = -5(2)^2 + 20(2) + 8$
   $y = 28$
   The vertex is (2, 28). The ball reaches its maximum height of 28 m in 2 s.

4a. Student C provided the better explanation. This student used complete sentences and gave specific information about the solution of the problem.

b. I would advise student D to give more details to ensure that the answer could be found using his or her explanation. Also, I would advise student D to mention graphing the function and what the zeros of the function relate to.

5. The response should include an explanation of how to determine the height and base of the figure. Units should be included. Use the area formula to write an equation.
   $A = bh = (x + 12)x = 13 \rightarrow x^2 + 12x = 13$. Solve for x, the height, by completing the square.
   $x^2 + 12x + 36 = 49$
   $(x + 6)^2 = 49$
   $x + 6 = \pm\sqrt{49}$
   $x + 6 = \pm 7$; $x = 1$ or $x = -13$
   The height is 1 cm and the base is $x + 12$, or 13 cm. The value of -13 for x can be eliminated because the height cannot be negative.

Answers to Test Items

A. yes; D: $0 \le x \le 2$; R: $0 \le y \le 10$
B. 28 m; 2 s
C. 6 s
D. $h = 1$ cm; $b = 13$ cm

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Organizer

Objective: Provide review and practice for this chapter and standardized tests.

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Chapter Cumulative Test

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CUMULATIVE ASSESSMENT

Multiple Choice

1. Which expression is NOT equal to the other three?

Ⓐ 0^1 Ⓒ 1^0

Ⓑ 1^1 Ⓓ $(-1)^0$

2. Which function's graph is a translation of the graph of $f(x) = 3x^2 + 4$ seven units down?

Ⓕ $f(x) = -4x^2 + 4$

Ⓖ $f(x) = 10x^2 + 4$

Ⓗ $f(x) = 3x^2 - 3$

Ⓙ $f(x) = 3x^2 + 11$

3. The area of a circle in square units is $\pi(9x^2 + 42x + 49)$. Which expression represents the circumference of the circle in units?

Ⓐ $\pi(3x + 7)$

Ⓑ $2\pi(3x + 7)$

Ⓒ $2\pi(3x + 7)^2$

Ⓓ $6x + 14$

4. CyberCafe charges a computer station rental fee of \$5, plus \$0.20 for each quarter-hour spent surfing. Which expression represents the total amount Carl will pay to use a computer station for three and a half hours?

Ⓕ $5 + 0.20(3.5)$

Ⓖ $5 + 0.20(3.5)(4)$

Ⓗ $5 + \frac{0.20}{3.5 \div 4}$

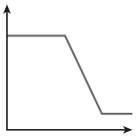
Ⓙ $5 + \frac{1}{4} \cdot \frac{0.20}{3.5}$

5. What is the numerical solution to the equation *five less than three times a number equals four more than eight times the number?*

Ⓐ $-\frac{9}{5}$ Ⓒ $\frac{1}{11}$

Ⓑ $-\frac{1}{5}$ Ⓓ $\frac{1}{5}$

6. Which is a possible situation for the graph?



Ⓕ A car travels at a steady speed, slows down in a school zone, and then resumes its previous speed.

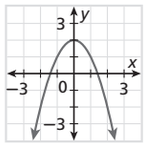
Ⓖ A child climbs the ladder of a slide and then slides down.

Ⓗ A person flies in an airplane for a while, parachutes out, and gets stuck in a tree.

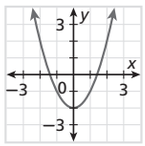
Ⓙ The number of visitors increases in the summer, declines in the fall, and levels off in the winter.

7. Which of the following is the graph of $f(x) = -x^2 + 2$?

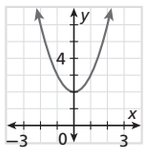
Ⓐ



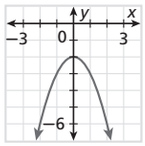
Ⓒ



Ⓑ



Ⓓ



8. The value of y varies directly with x, and $y = 40$ when $x = -5$. Find y when $x = 8$.

Ⓕ 25 Ⓗ -8

Ⓖ -1 Ⓙ -64

9. What is the slope of the line that passes through the points $(4, 7)$ and $(5, 3)$?

Ⓐ 4 Ⓒ $-\frac{1}{4}$

Ⓑ $\frac{1}{4}$ Ⓓ -4

Answers

1. A
2. H
3. B
4. G
5. A
6. H
7. A
8. G
9. D
10. H
11. C
12. G
13. C
14. H
S1. 1

The problems on many standardized tests are ordered from least to most difficult, but all items are usually worth the same amount of points. If you are stuck on a question near the end of the test, your time may be better spent rechecking your answers to earlier questions.

10. Putting Green Mini Golf charges a $4 golf club rental fee plus $1.25 per game. Good Times Golf charges a $1.25 golf club rental fee plus $3.75 per game. Which system of equations could be solved to determine for how many games the cost is the same at both places?

Ⓕ $\begin{cases} y = 4x + 1.25 \\ y = 3.75 + 1.25x \end{cases}$

Ⓖ $\begin{cases} y = 4 - 1.25x \\ y = -3.75 + 1.25x \end{cases}$

Ⓗ $\begin{cases} y = 1.25x + 4 \\ y = 3.75x + 1.25 \end{cases}$

Ⓙ $\begin{cases} y = 1.25x - 4 \\ y = 1.25x + 3.75 \end{cases}$

11. The graph of which quadratic function has an axis of symmetry of $x = -2$?

Ⓐ $y = 2x^2 - x + 3$

Ⓑ $y = 4x^2 + 2x + 3$

Ⓒ $y = x^2 - 2x + 3$

Ⓓ $y = x^2 + 4x + 3$

12. Which polynomial is the product of $x - 4$ and $x^2 - 4x + 1$?

Ⓕ $-4x^2 + 17x - 4$

Ⓖ $x^3 - 8x^2 + 17x - 4$

Ⓗ $x^3 + 17x - 4$

Ⓙ $x^3 - 15x + 4$

Gridded Response

13. The length of a rectangle is 2 units greater than the width. The area of the rectangle is 24 square units. What is its width in units?

14. Find the value of the discriminant of the equation $0 = -2x^2 + 3x + 4$.

15. What is the positive solution of $4x^2 = 10x + 2$? Round your answer to the nearest hundredth if necessary.

Short Response

16. The data in the table shows ordered pair solutions to a linear function. Find the missing y-value. Show your work.

x	y
-2	-7
-1	-3
0	
1	5
2	9

17. Answer the following questions using the function $f(x) = 2x^2 + 4x - 1$.

a. Make a table of values and give five points on the graph.

b. Find the axis of symmetry and vertex. Show all calculations.

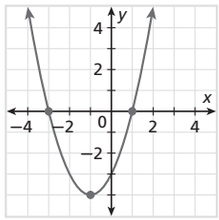
18. a. Show how to solve $x^2 - 2x - 8 = 0$ by graphing the related function. Show all your work.

b. Show another way to solve the equation in part **a**. Show all your work.

19. What can you say about the value of a if the graph of $y = ax^2 - 8$ has no x-intercepts? Explain.

Extended Response

20. The graph shows the quadratic function $f(x) = ax^2 + bx + c$.



a. What are the solutions of the equation $0 = ax^2 + bx + c$? Explain how you know.

b. If the point $(-5, 12)$ lies on the graph of $f(x)$, the point $(a, 12)$ also lies on the graph. Find the value of a.

c. What do you know about the relationship between the values of a and b? Use the coordinates of the vertex in your explanation.

d. Use what you know about solving quadratic equations by factoring to make a conjecture about the values of a, b, and c in the function $f(x) = ax^2 + bx + c$.

Short-Response Rubric

Items 16–19

2 Points = The student's answer is an accurate and complete execution of the task or tasks.

1 Point = The student's answer contains attributes of an appropriate response but is flawed.

0 Points = The student's answer contains no attributes of an appropriate response.

Extended-Response Rubric

Item 20

4 Points = The student writes the correct solutions and an explanation in part **a,** gives a correct x-value and correct explanation in part **b,** explains the relationship between a and b and uses the vertex in the explanation in part **c,** and uses solutions to find factors and the values for a, b, and c in part **d.**

3 Points = The student writes the correct solutions and an explanation in part **a,** gives a correct x-value and some explanation in part **b,** explains the relationship between a and b with minor errors and uses the vertex in the explanation in part **c,** and uses solutions to find factors and the values for a, b, and c in part **d.**

2 Points = The student writes one correct solution with some explanation in part **a,** gives a correct x-value and some explanation in part **b,** explains the relationship between a and b with minor errors and does not use the vertex in the explanation in part **c,** and uses solutions to find only one factor and does not find the values for a, b, and c in part **d;** or the student attempts to answer all parts of the problem and answers two correctly.

1 Point = The student answers one part correctly but does not attempt all parts, or the student attempts to answer all parts of the problem but does not correctly answer any part.

0 Points = The student does not answer correctly and does not attempt all parts of the problem.

Answers

S2a. Possible answer:

x	-3	-1	0	1	2
y	5	-3	-1	5	15

b. $x = -1$; $(-1, -3)$

S3a. -2, 4

b. Check students' work.

S4. $a < 0$

E1a. -3, 1

b. $a = 3$

c. $b = 2a$

d. If the solutions are -3 and 1, then $(x + 3)(x - 1) = 0$. So the equation is $x^2 + 2x - 3 = 0$, and $a = 1$, $b = 2$, and $c = -3$.